Kinematics

A particle moves along a straight line: its displacement s, its velocity v, its acceleration a are three functions of time, each the derivative of the one before. The exam asks when it is at rest, which way it moves, whether it speeds up, how far it goes. Each of these has its own test, and its own trap.

I DISPLACEMENT, VELOCITY, ACCELERATION

SL 5.9 · $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$

I.1 v and a from s

Definition 1 *displacement, velocity, acceleration*

$s(t)$: the **displacement** from a fixed point O (m, signed). **Velocity** $v(t) = s'(t)$ ($\mathrm{m\,s^{-1}}$); **acceleration** $a(t) = v'(t) = s''(t)$ ($\mathrm{m\,s^{-2}}$).

Example 1

A particle has displacement $s(t) = t^3 - 4t^2 + 5t$ metres. Find its velocity and acceleration at $t = 2$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $v = s'(t)$	$v(t) = 3t^2 - 8t + 5$
Step 2 $a = v'(t)$	$a(t) = 6t - 8$
Step 3 Substitute $t = 2$, with units	$v(2) = 1\ \mathrm{m\,s^{-1}},\ a(2) = 4\ \mathrm{m\,s^{-2}}$

APPLY

1 $s = 7t - t^2$: $v(t)$ and $a(t)$.

2 $s = 2t^3$: $v(1)$ and $a(1)$.

I.2 At rest, initially

Property 1 *at rest, initially*

At rest: $v(t) = 0$. "Initially": $t = 0$, the initial displacement $s(0)$, the initial velocity $v(0)$.

Watch out 1

Displacement is not distance. And "at rest" is $v = 0$, not $s = 0$ (that is "at O").

Example 2

$s(t) = t^3 - 12t^2 + 45t$, $t \geq 0$. Find the initial velocity, the times at rest, and the acceleration the second time it is at rest.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Initially: $v(0)$	$v(t) = 3t^2 - 24t + 45,\ v(0) = 45\ \text{m s}^{-1}$
Step 2 At rest: $v(t) = 0$	$3(t-3)(t-5) = 0:\ t = 3, t = 5$
Step 3 a at the second time	$a(t) = 6t - 24,\ a(5) = 6\ \text{m s}^{-2}$

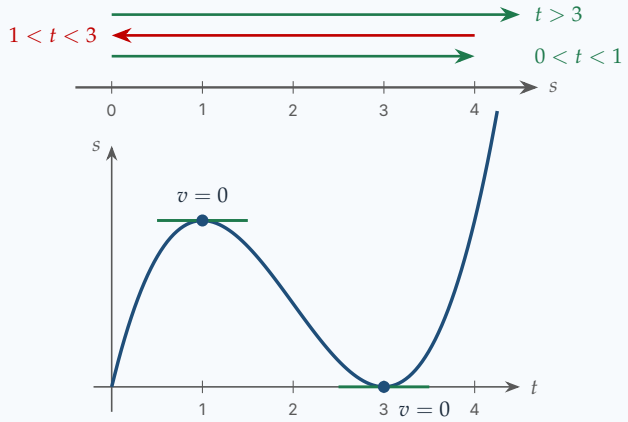


Figure 1 — $s(t) = t^3 - 6t^2 + 9t$: the particle stops at $t = 1$ and $t = 3$, the horizontal tangents of the graph of s.

APPLY

3 $s = t^2 - 10t + 3$: when at rest, and the initial velocity.

4 $v = 2\cos t$: the first time at rest.

II DIRECTION, SPEEDING UP AND SLOWING DOWN

SL 5.9 · the signs of v and of a

II.1 Direction of motion

Property 2 *direction*

Positive direction when $v(t) > 0$, negative when $v(t) < 0$. It **changes direction** where v changes sign.

Example 3

$v(t) = t^2 - 7t + 10$, $t \geq 0$. When does the particle move in the negative direction, and when does it change direction?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Factorise v	$(t-2)(t-5)$
Step 2 Sign of v	negative for $2 < t < 5$
Step 3 Changes of sign	at $t = 2$ and $t = 5$

APPLY

1 $v = 8 - 2t$: when does it move in the positive direction?

2 $v = 4\sin t$, $0 \leq t \leq 2\pi$: when in the negative direction?

II.2 Speeding up or slowing down

Property 3 *speed*

Speed $|v(t)|$. **Speeding up** when v and a have the **same sign**; **slowing down** when they have opposite signs.

Watch out 2

$a < 0$ does not mean slowing down: with $v < 0$ too, the particle goes faster. Compare the two signs.

Example 4

$v(t) = t^2 - 6t + 8$, $t \geq 0$. Find when the particle is slowing down.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Signs of $v = (t-2)(t-4)$	$+$ for $t < 2$, $-$ for $2 < t < 4$, $+$ for $t > 4$
Step 2 Signs of $a = 2t - 6$	$-$ for $t < 3$, $+$ for $t > 3$
Step 3 Opposite signs	$0 \leq t < 2$ and $3 < t < 4$

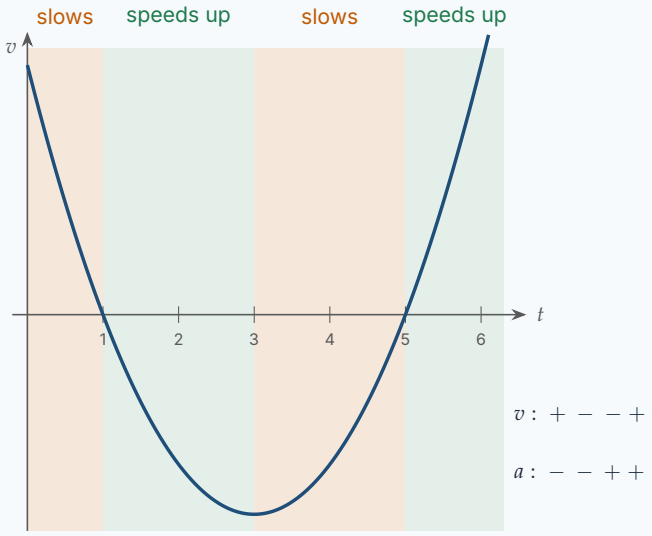


Figure 2 — $v(t) = 3(t-1)(t-5)$, $a(t) = 6t - 18$: slowing down where v and a disagree.

APPLY

3 At an instant $v = -4$ and $a = -2$: speeding up or slowing down?

4 At an instant $v = 3$ and $a = -1$?

III.1 Distance travelled, displacement

Property 4 *distance travelled*

Distance travelled: add the lengths of the legs between changes of direction, $\int_{t_1}^{t_2} |v|\,\mathrm{d}t$ on the GDC. **Displacement**: $s(t_2) - s(t_1) = \int_{t_1}^{t_2} v\,\mathrm{d}t$.

Example 5

$s(t) = 10t - t^2, 0 \le t \le 8$. Find the displacement and the distance travelled.

WHAT YOU DO	WHAT YOU WRITE
Step 1 When does it turn? $v = 0$	$10 - 2t = 0$: $t = 5$
Step 2 Positions at 0, 5, 8	$s(0) = 0,\ s(5) = 25,\ s(8) = 16$
Step 3 Displacement: end minus start	16 m
Step 4 Distance: add the legs	$25 + 9 = 34$ m

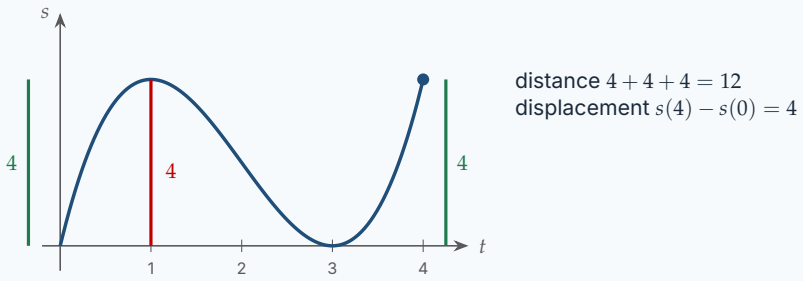


Figure 3 — $s(t) = t^3 - 6t^2 + 9t$ from 0 to 4: displacement 4 m, distance 12 m.

APPLY

1 $s = t^2 - 2t, 0 \le t \le 4$: distance and displacement.

III.2 The greatest speed

Property 5 *greatest speed*

The greatest **speed** is the largest $|v|$: compare the maximum of v, the minimum of v (its size) and the ends.

Watch out 3

A velocity of $-16\ \text{m s}^{-1}$ is a speed of 16, faster than 9. Distance needs $|v|$; $\int v$ gives the displacement.

Example 6

$v(t) = 6t - t^2, 0 \leq t \leq 8$. Find the greatest speed.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Extremum of v: $a = 0$	$t = 3,\ v = 9$
Step 2 The ends	$v(0) = 0,\ v(8) = -16$
Step 3 Largest size	$16\ \text{m s}^{-1}$, at $t = 8$

APPLY

2 $v = t^2 - 4, 0 \leq t \leq 3$: the greatest speed.

III.3 On the GDC

Example 7

$v(t) = 4\cos t - t, 0 \leq t \leq 4$. Find when the particle is at rest, the distance travelled and the displacement.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Zero of v on the GDC	$t = 1.25$
Step 2 Distance: $\int \lvert v \rvert$	$\int_0^4 \lvert v \rvert \, \mathrm{d}t = 17.1\ \text{m}$
Step 3 Displacement: $\int v$	$-11.0\ \text{m}$

APPLY

3 Same v: the velocity at $t = 4$.

IV EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A s, v, a: at rest

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 6]

A particle moves along a straight line. Its displacement from a fixed point O, s metres, at time t seconds is $s(t) = 2t^3 - 15t^2 + 36t$, for $t \geq 0$.

(a) Find an expression for the velocity $v(t)$. [2]

(b) Find the times at which the particle is at rest. [2]

(c) Find the acceleration of the particle when it is first at rest. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $v = s'(t)$	$v(t) = 6t^2 - 30t + 36$	A2
	A1 for two correct terms.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 At rest: factorise v	$6(t-2)(t-3) = 0$	M1
	At rest: $v = 0$.	
Step 2 Solve	$t = 2$ s and $t = 3$ s	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $a = v'(t)$	$a(t) = 12t - 30$	A1
Step 2 First time at rest	$a(2) = -6\ \text{m s}^{-2}$	A1
	"First at rest": $t = 2$, the smaller time.	

EXAM-STYLE QUESTION 2

Paper 1 · no calculator · [Maximum mark: 5]

The velocity of a particle, v m s^{-1}, at time t seconds is $v(t) = 4\sin\left(\frac{\pi t}{6}\right)$, for $0 \leq t \leq 12$.

(a) Find the acceleration $a(t)$. [2]

(b) Find the exact value of the acceleration when $t = 1$. [2]

(c) Find the value of t, with $0 < t < 12$, at which the particle is at rest. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $a = v'(t)$, chain rule	$a(t) = 4 \times \frac{\pi}{6}\cos\left(\frac{\pi t}{6}\right) = \frac{2\pi}{3}\cos\left(\frac{\pi t}{6}\right)$	A1A1
	A1 for cos, A1 for the factor $\frac{\pi}{6}$ from the chain rule.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute $t = 1$	$a(1) = \frac{2\pi}{3}\cos\frac{\pi}{6} = \frac{2\pi}{3} \times \frac{\sqrt{3}}{2}$	M1
Step 2 Exact value	$= \frac{\pi\sqrt{3}}{3}$ m s^{-2}	A1
	Accept $\frac{\pi}{\sqrt{3}}$.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 At rest: $v = 0$	$t = 6$	A1
	$\sin\left(\frac{\pi t}{6}\right) = 0$ *with* $0 < t < 12$: $\frac{\pi t}{6} = \pi$.	

EXAM-STYLE QUESTION 3

Paper 1 · no calculator · [Maximum mark: 6]

A particle moves along a straight line so that its displacement, s metres, at time t seconds is $s(t) = t^3 - 9t^2 + 15t$, for $t \geq 0$.

(a) Find $v(t)$ and $a(t)$. [2]

(b) Find the values of t for which the particle moves in the negative direction. [2]

(c) Find the values of t for which the particle is slowing down. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $v = s'$, factorised	$v(t) = 3t^2 - 18t + 15 = 3(t-1)(t-5)$	A1
Step 2 $a = v'$	$a(t) = 6t - 18$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Negative direction: $v < 0$	$v(t) < 0$	M1
Step 2 Write the interval	$1 < t < 5$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Opposite signs of v and a	v and a of opposite signs *A sign table with three lines: t, v, a.*	M1
Step 2 Write the intervals	$0 \leq t < 1$ and $3 < t < 5$ *Both intervals for A1.*	A1

EXAM-STYLE QUESTION 4

Paper 1 · no calculator · [Maximum mark: 6]

A particle moves along a line with displacement $s(t) = t^2 - 6t + 5$ metres from O, for $0 \le t \le 5$, t in seconds.

(a) Find the time at which the particle changes direction. [2]

(b) Find the displacement of the particle at $t = 5$ relative to its starting position. [1]

(c) Find the total distance travelled by the particle in the first 5 seconds. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Changes direction: $v = 0$	$v(t) = 2t - 6 = 0$	M1
Step 2 Solve	$t = 3$ s	A1
	v changes sign there: from $-$ to $+$.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 End minus start	$s(5) - s(0) = 0 - 5 = -5$ m	A1
	Displacement: from the start to the end, with its sign.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Positions at the turning time	$s(0) = 5,\ s(3) = -4,\ s(5) = 0$	(M1)
	The positions at the start, at the turning time, at the end.	
Step 2 Add the legs	$\lvert s(3) - s(0)\rvert + \lvert s(5) - s(3)\rvert = 9 + 4$	M1
	Distance: each leg counted positive, split at the change of direction.	
Step 3 Compute	13 m	A1

EXAM-STYLE QUESTION 5

Paper 1 · no calculator · [Maximum mark: 5]

The velocity of a particle, v m s^{-1}, at time t seconds is $v(t) = t^2 - 4t - 5$, for $0 \le t \le 7$.

(a) Find the least value of v. [2]

(b) Find the greatest speed of the particle for $0 \le t \le 7$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Extremum of v: $a = 0$	$a(t) = 2t - 4 = 0,\ t = 2$	M1
Step 2 The least velocity	$v(2) = -9$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Compare the sizes with the ends	compare $\lvert v(2)\rvert = 9$ with the ends: $\lvert v(0)\rvert = 5$, $\lvert v(7)\rvert = 16$	M1
	Speed is $\lvert v\rvert$: the end $t = 7$ wins.	
Step 2 The greatest speed	16 m s^{-1}	A2
	9 (the size of the least velocity) earns M1A0.	

C On the GDC (Paper 2)

EXAM-STYLE QUESTION 6 — Paper 2 · calculator · [Maximum mark: 7]

A particle moves in a straight line with velocity $v(t) = 6\cos(0.9t)\,\mathrm{e}^{-0.2t}$ m s^{-1}, for $0 \le t \le 8$, where t is in seconds.

(a) Find the times at which the particle is at rest. [2]

(b) Find the greatest speed of the particle. [2]

(c) Find the total distance travelled by the particle. [2]

(d) Find the displacement of the particle at $t = 8$ from its position at $t = 0$. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Zeros of v on the GDC	$t = 1.75$ $(1.74532\ldots)$ *Exact:* $\cos(0.9t) = 0$, $t = \frac{\pi}{1.8}$.	A1
	$t = 5.24$ $(5.23598\ldots)$ *The next zero, 8.73, is outside the domain.*	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Compare the extrema and the ends	compare $v(0) = 6$ with the minimum $v = -3.06$ *Speed is $\lvert v\rvert$: the minimum of v is a candidate too.*	M1
Step 2 The greatest speed	6 m s^{-1} (at $t = 0$)	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Distance: $\int \lvert v\rvert$	$\displaystyle\int_0^8 \lvert v(t)\rvert\,\mathrm{d}t$ *The absolute value: without it, the GDC gives the displacement.*	(M1)
Step 2 Compute	15.7 m $(15.6771\ldots)$	A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Displacement: $\int v$	$\displaystyle\int_0^8 v(t)\,\mathrm{d}t = 2.26$ m $(2.25635\ldots)$	A1

EXAM-STYLE QUESTION 7 Paper 2 · calculator · [Maximum mark: 6]

The velocity of a particle, v m s^{-1}, at time t seconds is $v(t) = 2t - 5\ln(t+1)$, for $0 \le t \le 10$.

(a) Find the acceleration $a(t)$. [2]

(b) Find the least velocity of the particle. [2]

(c) Find the value of $t > 0$ at which the particle is at rest. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $a = v'(t)$	$a(t) = 2 - \dfrac{5}{t+1}$	A1A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Minimum of v: $a = 0$	$a(t) = 0$ at $t = 1.5$	M1
Step 2 Compute	$v(1.5) = 3 - 5\ln 2.5 = -1.58 \ \ (-1.58145\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write $v(t) = 0$	$v(t) = 0$ (seen or implied by a sketch) *$t = 0$ is a solution too: the question asks for $t > 0$.*	(M1)
Step 2 Solve on the GDC	$t = 4.05 \ \ (4.04697\ldots)$	A1

V WHICH METHOD? READ THE QUESTION

5.9 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"find the velocity / the acceleration"	s given	$v = s'(t)$, $a = v'(t) = s''(t)$, with units D1
"at rest"	$v = 0$	solve $v(t) = 0$ in the domain; "first at rest": the smallest $t > 0$ P1
"initially"	$t = 0$	$s(0)$, $v(0)$, $a(0)$ P1
"moves in the positive direction", *"changes direction"*	the sign of v	$v > 0$, or the times where v changes sign P2
"speeding up", *"slowing down"*	v and a	same signs: speeding up; opposite: slowing down. A sign table P3
"total distance travelled"	changes of direction	Paper 1: split at the turning times and add $\lvert\Delta s\rvert$; Paper 2: $\int \lvert v\rvert \, dt$ on the GDC P4
"greatest speed"	$\lvert v\rvert$	max of v, min of v (its size), the ends: the largest P5
Paper 2, v with sin, e *or* ln	the GDC is allowed	Part V: zeros of v, extrema of v, $\int \lvert v\rvert$

In the exam 1 *command terms*

At rest: $v = 0$. **Initially**: $t = 0$. **Speed**: $|v|$, never negative. **Displacement**: a signed position from O, or a change of position. **Distance travelled**: always positive, every leg counted. Units: m, $\mathrm{m\,s^{-1}}$, $\mathrm{m\,s^{-2}}$.

VI ON THE GDC: PAPER 2

SL 5.9 · NumWorks · every calculator move of this topic

1 When is the particle at rest?

Grapher → v(x) → Find → Zeros

Enter v with x for t, the window on the domain, then the zeros of v. **On paper:** "$v(t) = 0$", then each time.

GRAPHER · ZEROS

```
f(x)=6cos(0.9x)e^(-0.2x)
x=1.745329 y=0
x=5.235988 y=0
```

2 The greatest speed

Grapher → Find → Maximum → Minimum

Both the maximum and the minimum of v, and the values at the ends: the largest *size* is the greatest speed.

GRAPHER · MINIMUM

```
x=3.247693 y=-3.059097
v(0)=6
greatest speed: 6
```

3 The distance travelled

Grapher → f(x) = abs(v(x)) → Find → Integral

Integrate $|v|$, not v, between the two times. **On paper:** $\displaystyle\int_0^8 |v(t)|\,\mathrm{d}t = 15.7$ m.

GRAPHER · INTEGRAL

```
f(x)=abs(6cos(0.9x)
*e^(-0.2x))
from 0 to 8
integral = 15.67715
```

ANSWERS TO THE APPLY QUESTIONS

I.1 $7-2t;\ -2$

I.2 $6\ \mathrm{m\,s^{-1}};\ 12\ \mathrm{m\,s^{-2}}$

I.3 $t=5;\ -10\ \mathrm{m\,s^{-1}}$

I.4 $t=\frac{\pi}{2}$

II.1 $0\le t<4$

II.2 $\pi<t<2\pi$

II.3 speeding up

II.4 slowing down

III.1 $10\ \mathrm{m};\ 8\ \mathrm{m}$

III.2 $5\ \mathrm{m\,s^{-1}}$

III.3 $-6.61\ \mathrm{m\,s^{-1}}$

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