Kinematics

Skills sheet · SL AA 5.9

KEY FACTS

$v = s'(t)$, $a = v'(t) = s''(t)$ D1
At rest: $v = 0$; initially: $t = 0$ P1
Speeding up: v and a of the same sign P3
Distance: $\int |v|\,\mathrm{d}t$, or $|\Delta s|$ leg by leg P4

Displacement is not distance W1.
$a < 0$ is not "slowing down" W2.
Greatest speed: the largest $|v|$ W3.

WHAT UNLOCKS

- "at rest" $\rightarrow$ $v(t) = 0$, solved in the domain.
- "changes direction" $\rightarrow$ v changes sign there.
- "slowing down" $\rightarrow$ a sign table with v and a, intervals where they differ.
- "distance travelled" $\rightarrow$ the turning times first; each leg counted positive.

WORKED EXAMPLE ①

$s(t) = t^3 - 6t^2$, $t \geq 0$. When is the particle at rest, and what is its acceleration then? no GDC

$v(t) = 3t^2 - 12t = 3t(t-4) = 0$ M1
$t = 0$ and $t = 4$ A1
$a(t) = 6t - 12$: $a(0) = -12$, $a(4) = 12\ \mathrm{m\,s^{-2}}$ A1A1

Method
How do I read a motion?
① $v = s'$, $a = v'$, with units.
② Zeros of v: rest and turning.
③ Signs of v and a in a table.
④ Answer in t, inside the domain.
$s = 0$ means "at O", not "at rest".

WORKED EXAMPLE ②

$s(t) = t^2 - 4t$, $0 \leq t \leq 5$. Find the distance travelled. no GDC

$v = 2t - 4 = 0$ at $t = 2$: a turn (M1)
$s(0) = 0$, $s(2) = -4$, $s(5) = 5$ A1
$4 + 9 = 13$ m A1

Method
How do I find a distance?
① The times where $v = 0$ in the interval.
② s at the start, at each turn, at the end.
③ The size of each change of s.
④ Add them. Paper 2: $\int |v|$.

FLUENCY

●○○ **1.** $s = t^2 - 4t + 7$: find v and a. no GDC

●○○ **2.** $s = 5t - t^2$: when is the particle at rest? no GDC

●○○ **3.** $s = t^3 - 3t$: the initial velocity. no GDC

●○○ **4.** $v = 3\sin 2t$: find a. no GDC

●○○ **5.** $v = t^2 - 9$, $t \geq 0$: when does it move in the negative direction? no GDC

●○○ **6.** $v = -4$ and $a = -2$: speeding up or slowing down? no GDC

REASONING

●●● **Question 1.** A particle has displacement $s(t) = t^3 - 12t^2 + 36t$ metres, $t \geq 0$ seconds. [no GDC]

(a) Find the times at which it is at rest. [3]

(b) Find when it moves in the negative direction. [1]

(c) Find when it is slowing down. [3]

●●● **Question 2.** A particle has velocity $v(t) = (2 - t)\,\mathrm{e}^{-t}\ \mathrm{m\,s^{-1}}$, $t \geq 0$. [no GDC]

(a) Find when it is at rest. [1]

(b) Show that $a(t) = (t - 3)\,\mathrm{e}^{-t}$, and find the least velocity. [4]

●●● **Question 3.** A particle has displacement $s(t) = t^2 - 8t + 12$ metres, for $0 \leq t \leq 6$. [no GDC]

(a) Find the total distance travelled. [3]

(b) Find the displacement over the six seconds. [1]

PROBLEMS

●●● **Question 4.** Two particles A and B move on the same line: $s_A(t) = t^2 - 4t$ and $s_B(t) = 2t - t^2$, $t \geq 0$. [no GDC]

(a) Find the time at which they have the same velocity. [2]

(b) Find the times at which they are at the same position. [3]

●●● **Question 5.** A particle has velocity $v(t) = 12 - 3t^2\ \mathrm{m\,s^{-1}}$, for $0 \leq t \leq 3$. [no GDC]

(a) Find the greatest speed. Justify your answer. [3]

(b) Find the values of t for which the particle speeds up. [3]

ANSWERS

1 $v = 2t - 4,\ a = 2$

2 $5 - 2t = 0$: $t = 2.5$

3 $v = 3t^2 - 3$: $v(0) = -3\ \text{m s}^{-1}$

4 $a = 6\cos 2t$

5 $0 \le t < 3$

6 same signs: speeding up

Q1 (a) $v = 3(t-2)(t-6)$: $t = 2, t = 6$ (b) $2 < t < 6$ (c) $a = 6t - 24$: slowing down for $0 \le t < 2$ and $4 < t < 6$

Q2 (a) $t = 2$ (b) $-\text{e}^{-t} - (2-t)\text{e}^{-t} = (t-3)\text{e}^{-t}$; least velocity at $t = 3$: $v(3) = -\text{e}^{-3}$

Q3 (a) turn at $t = 4$: $s(0) = 12, s(4) = -4, s(6) = 0$: $16 + 4 = 20$ m (b) $s(6) - s(0) = -12$ m

Q4 (a) $2t - 4 = 2 - 2t$: $t = 1.5$ (b) $t^2 - 4t = 2t - t^2, 2t(t-3) = 0$: $t = 0$ and $t = 3$

Q5 (a) $v(0) = 12, v(3) = -15$, v decreasing: greatest speed $15\ \text{m s}^{-1}$ at $t = 3$ (b) $a = -6t \le 0$; $v < 0$ for $t > 2$: speeding up for $2 < t \le 3$

Written and verified by a practising IB mathematics teacher, produced with AI assistance; every result recomputed by software, every figure computed from its function. Not affiliated with or endorsed by the International Baccalaureate Organization. © 2026 Hence