Marked example 1

Paper 2 style · Graphic display calculator

1 6 marks SL 5.8 ●●● Demanding

A small ceramics workshop makes and sells x mugs per week. Its weekly profit, in Swiss francs, is modelled by

$$P(x) = -0.02x^3 + 1.5x^2 + 12x - 400, \qquad 0 \le x \le 80.$$

(a) Find the weekly profit when the workshop makes 20 mugs. **[1]**

(b) Find the value of x for which $P'(x) = 0$. **[2]**

The workshop can only make a whole number of mugs.

(c) Find the number of mugs the workshop should make each week to maximize its profit, and find this maximum profit, to the nearest franc. **[3]**

MARKSCHEME (a)

$P(20) = 280$ francs — **A1**

Total 1 mark

MARKSCHEME (b)

$P'(x) = -0.06x^2 + 3x + 12 = 0$, or the maximum of P on the GDC — **(M1)**

$x = 53.7 \ \ (53.7228\ldots)$ — **A1**

Total 2 marks

Note The negative root -3.72 is outside the domain and must not be given as the answer.

MARKSCHEME (c)

comparing the two whole numbers either side: $P(53) = 1471.96$ and $P(54) = 1472.72$ — **M1**

54 mugs — **A1**

maximum profit 1473 francs — **A1**

Total 3 marks

Note Rounding 53.7 to 54 without comparing $P(53)$ and $P(54)$ earns M0A1A1. An answer of 1473 from $P(53.7) = 1472.85$ earns A0 for the profit: that number of mugs is not possible.

ON THE CALCULATOR

NumWorks Functions: $f(x) = -0.02x^3 + 1.5x^2 + 12x - 400$; Axes: x from 0 to 80, y from -500 to 1600. Graph → Calculate → Find → Maximum → $x = 53.72281$. Then Table: $f(53) = 1471.96$, $f(54) = 1472.72$.

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