Marked example 2

Paper 1 style · No calculator

2 14 marks SL 5.8

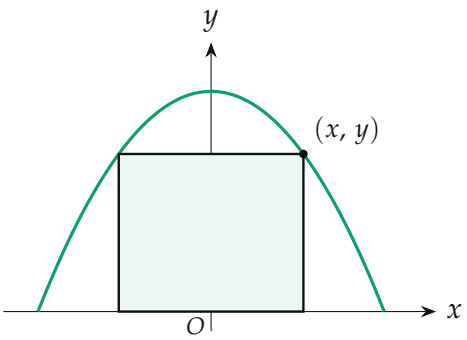
●●○ Typical to demanding

The entrance to a tunnel is an arch whose shape is modelled by

$$y = 9 - \frac{x^2}{4},$$

where x and y are measured in metres and the x-axis is the ground.

A rectangular door is fitted inside the arch, with its base on the ground and its two top corners on the arch, as shown in the diagram. The point (x, y) is the top right-hand corner of the door, where $0 < x < 6$.



(a) Find the width of the arch at ground level. [2]

(b) Show that the area A of the door, in square metres, is given by $A = 18x - \frac{x^3}{2}$. [2]

(c) Find $\frac{dA}{dx}$. [2]

(d) Find the exact value of x for which the area of the door is a maximum. Justify that it is a maximum. [4]

(e) Find the exact maximum area of the door. [2]

(f) Show that the door of maximum area has a height equal to two thirds of the height of the arch. [2]

MARKSCHEME (a)

$9 - \frac{x^2}{4} = 0 \Rightarrow x = \pm 6$ (M1)

width $= 12$ m A1

Total 2 marks

MARKSCHEME (b)

width of door $= 2x$, height $= 9 - \frac{x^2}{4}$ M1

$A = 2x\left(9 - \frac{x^2}{4}\right)$ A1

$A = 18x - \frac{x^3}{2}$ AG

Total 2 marks

MARKSCHEME (c)

$\frac{dA}{dx} = 18 - \frac{3x^2}{2}$ A1A1

Total 2 marks

MARKSCHEME (d)

$18 - \frac{3x^2}{2} = 0$ — M1

$x^2 = 12$ — (A1)

$x = 2\sqrt{3}$ (reject $x = -2\sqrt{3}$ since $x > 0$) — A1

$\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = -3x < 0$ at $x = 2\sqrt{3}$, so maximum — R1

Total 4 marks

Note Accept $\sqrt{12}$. Accept a sign change of $\frac{\mathrm{d}A}{\mathrm{d}x}$ for R1.

MARKSCHEME (e)

$A = 18(2\sqrt{3}) - \frac{(2\sqrt{3})^3}{2} = 36\sqrt{3} - 12\sqrt{3}$ — (M1)

$A = 24\sqrt{3}\ \mathrm{m}^2$ — A1

Total 2 marks

MARKSCHEME (f)

height of door $= 9 - \frac{12}{4} = 6$ m; height of arch $= 9$ m — A1

$\frac{6}{9} = \frac{2}{3}$ — R1 AG

Total 2 marks

Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software.
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