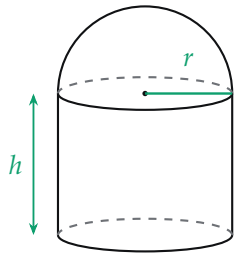
Marked example 3

Paper 2 style · Graphic display calculator

3 15 marks SL 5.8

●●○ Typical to demanding

A sensor housing is made from thin sheet metal. It consists of a cylinder of radius r cm and height h cm, closed at the base, with a hemispherical cap of radius r cm on top, as shown in the diagram. There is no metal between the cylinder and the cap.
The volume enclosed by the housing is 500 cm^3.



(a) Show that $h = \frac{500}{\pi r^2} - \frac{2r}{3}$. [2]

(b) Show that the total external surface area, S cm^2, of the housing is given by $S = \frac{5}{3}\pi r^2 + \frac{1000}{r}$. [3]

(c) Find $\frac{\mathrm{d}S}{\mathrm{d}r}$. [2]

(d) Find the value of r for which S is a minimum. Justify that it is a minimum. [3]

(e) Show that, for this value of r, the height of the cylinder is equal to its radius. [2]

The housing must fit inside a tube, so that its radius can be at most 4 cm.

(f) Find the least possible surface area of a housing that fits inside the tube. Justify your answer. [3]

MARKSCHEME (a)

$\pi r^2 h + \frac{2}{3}\pi r^3 = 500$ M1A1

$h = \frac{500}{\pi r^2} - \frac{2r}{3}$ AG

Total 2 marks

Note M1 for adding the volumes of a cylinder and of a hemisphere; A1 for the correct equation. The AG line must follow from correct working.

MARKSCHEME (b)

$S = \pi r^2 + 2\pi rh + 2\pi r^2$ (base, curved surface, hemisphere) M1A1

$2\pi r\left(\frac{500}{\pi r^2} - \frac{2r}{3}\right) = \frac{1000}{r} - \frac{4}{3}\pi r^2$ A1

$S = \frac{5}{3}\pi r^2 + \frac{1000}{r}$ AG

Total 3 marks

Note M1 for a sum of three areas; A1 for all three correct, including $2\pi r^2$ for the hemisphere.

MARKSCHEME (c)

$\frac{\mathrm{d}S}{\mathrm{d}r} = \frac{10}{3}\pi r - \frac{1000}{r^2}$ A1A1

Total 2 marks

MARKSCHEME (d)

$\frac{10}{3}\pi r - \frac{1000}{r^2} = 0$, or the minimum of S on the GDC **(M1)**

$r = 4.57 \ \ (4.57078\ldots)$ **A1**

EITHER

$\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = \frac{10}{3}\pi + \frac{2000}{r^3} > 0$ for $r > 0$, so minimum **R1**

OR

$\frac{\mathrm{d}S}{\mathrm{d}r}$ changes from negative to positive at $r = 4.57$, so minimum **R1**

Total 3 marks

Note Accept a GDC graph of S with a clearly identified minimum point for R1.

MARKSCHEME (e)

from (d), $\frac{10}{3}\pi r^3 = 1000$, so $\pi r^3 = 300$ **M1**

$h = \frac{500}{\pi r^2} - \frac{2r}{3} = \frac{500\,r}{\pi r^3} - \frac{2r}{3} = \frac{5r}{3} - \frac{2r}{3}$ **A1**

$h = r$ **AG**

Total 2 marks

Note A numerical check only (e.g. substituting $r = 4.57$ and obtaining $h \approx 4.57$) earns M0A0: the question asks for an exact argument.

MARKSCHEME (f)

$\frac{\mathrm{d}S}{\mathrm{d}r} < 0$ for $0 < r < 4.57$, so S is decreasing for $0 < r \leq 4$ **R1**

so the least value is at $r = 4$: $S = \frac{5}{3}\pi(16) + \frac{1000}{4}$ **M1**

$S = 334\ \text{cm}^2 \ \ (333.776\ldots)$ **A1**

Total 3 marks

Note Award R1 for any valid argument that the least value on $0 < r \leq 4$ occurs at $r = 4$, including a GDC graph of S restricted to this interval. An answer of 328 (the unconstrained minimum) earns R0M0A0.

ON THE CALCULATOR

NumWorks (d) Functions: $f(x) = \frac{5}{3}\pi x^2 + \frac{1000}{x}$. Graph → Axes → Values of X: 0 to 10; Values of Y: 0 to 600. Graph → Calculate → Find → Minimum → $x = 4.570781$, $f(x) = 328.1710$.

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