Exercise 2

Paper 1 style · No calculator

2 7 marks SL 5.8 ●●○ Typical

An open box, with no lid, has a square base of side x cm and height h cm. Its volume is 32 cm^3.

(a) Show that the area of card used to make the box is $S = x^2 + \frac{128}{x}$. **[2]**

(b) Find the dimensions of the box for which S is least. Justify your answer. **[4]**

(c) Find the least area of card. **[1]**

The markscheme is on the next page: try the question first. Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization.
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MARKSCHEME (a)

$x^2h = 32$, so $h = \frac{32}{x^2}$ **M1**

$S = x^2 + 4xh = x^2 + 4x \times \frac{32}{x^2} = x^2 + \frac{128}{x}$ **A1**

Total 2 marks

Note One base and four sides: no lid.

MARKSCHEME (b)

$\frac{dS}{dx} = 2x - \frac{128}{x^2} = 0$ **M1**

$x^3 = 64,\ x = 4$ **A1**

$\frac{d^2S}{dx^2} = 2 + \frac{256}{x^3} > 0$: a minimum **R1**

base 4 cm by 4 cm, height 2 cm **A1**

Total 4 marks

MARKSCHEME (c)

$S = 16 + 32 = 48$ cm^2 **A1**

Total 1 mark