Optimisation

Skills sheet · SL AA 5.8

KEY FACTS

$Q'(x) = 0$, justify, then answer the question P1
Two variables: the constraint, solved and substituted P2
Justify: Q'', or the sign of Q', or a sketch P3
Closed interval: the ends are candidates P1

"Maximum profit": a value of P, not of x W1.
"Show that $V = \ldots$": the constraint on the page W2.
Roots outside the domain are rejected, with a reason W3.

WHAT UNLOCKS

- a fixed perimeter, area or volume → the constraint: one variable in terms of the other.
- "possible values of x" → every length positive.
- "the least cost", "the largest area" → the value of the quantity, with units.
- a number of objects → whole numbers: compare the two either side.

WORKED EXAMPLE ①

40 m of fence makes three sides of a rectangle against a wall. Find the greatest area. no GDC

sides x, x, $40 - 2x$: $A = x(40 - 2x)$ — M1
$A'(x) = 40 - 4x = 0$: $x = 10$ — M1A1
$A'' = -4 < 0$: a maximum — R1
$A = 10 \times 20 = 200$ m^2 — A1

Method
How do I solve an optimisation?
① Name the variables; write the quantity.
② The constraint: down to one variable.
③ $Q'(x) = 0$, and the domain.
④ Justify; answer what is asked, with units.
Stopping at $x = 10$ loses the last A1.

WORKED EXAMPLE ②

Find the greatest value of $f(x) = x^3 - 3x^2$ for $-1 \leq x \leq 4$. no GDC

$f'(x) = 3x(x - 2) = 0$: $f(0) = 0$, $f(2) = -4$ — M1A1
ends: $f(-1) = -4$, $f(4) = 16$ — M1
greatest value 16, at $x = 4$ — A1

Method
Which value is the greatest?
① The stationary points in the interval.
② The values of f there.
③ The values of f at both ends.
④ The largest of all of them.

FLUENCY

●○○ **1.** Greatest value of $P(x) = -x^2 + 10x + 3$. no GDC

●○○ **2.** Least value of $C(x) = x^2 - 12x + 50$. no GDC

●○○ **3.** Greatest value of $A(x) = x(20 - x)$. no GDC

●○○ **4.** Least value of $x + \dfrac{16}{x}$, $x > 0$. no GDC

●○○ **5.** Two numbers have a sum of 20: their greatest product. no GDC

●○○ **6.** A rectangle has perimeter 40 cm: its greatest area. no GDC

REASONING

●●● **Question 1.** An open box is made from a square sheet of side 18 cm by cutting squares of side x cm from the corners. [no GDC]

(a) Write down the volume $V(x)$ and the possible values of x. [2]

(b) Find the greatest volume. Justify your answer. [4]

●●● **Question 2.** At a price of p francs, a club sells $200 - 5p$ tickets. [no GDC]

(a) Find the price that gives the greatest revenue, and this revenue. [4]

(b) The price may not exceed 15 francs. Find the greatest revenue now. [2]

●●● **Question 3.** An open cylindrical tank (no lid) has volume 27π m^3 and radius r m. [no GDC]

(a) Show that its area is $S = \pi r^2 + \dfrac{54\pi}{r}$. [2]

(b) Find the radius that makes S least, and the least area. [4]

PROBLEMS

●●● **Question 4.** Let $P\left(x, \sqrt{x}\right)$ be a point on the curve $y = \sqrt{x}$, and $A(4, 0)$. [no GDC]

(a) Show that $AP^2 = x^2 - 7x + 16$. [2]

(b) Find the least distance AP, as an exact value. [4]

●●● **Question 5.** A workshop makes n chairs a week, n a whole number. The weekly profit is $P(n) = -2n^2 + 49n - 100$ francs. [no GDC]

Find the number of chairs that gives the greatest profit, and this profit. Justify your answer. [5]

ANSWERS

1 $x = 5$: 28

2 $x = 6$: 14

3 $x = 10$: 100

4 $1 - \dfrac{16}{x^2} = 0$, $x = 4$: 8

5 $x(20 - x)$, $x = 10$: 100

6 $x(20 - x)$, a 10 by 10 square: $100\ \text{cm}^2$

Q1 (a) $V(x) = x(18 - 2x)^2$, $0 < x < 9$ (b) $V'(x) = (18 - 2x)(18 - 6x) = 0$, $x = 3$ ($x = 9$ rejected); sign of V' from $+$ to $-$: $V = 432\ \text{cm}^3$

Q2 (a) $R = 200p - 5p^2$, $R' = 200 - 10p = 0$: $p = 20$, $R = 2000$ francs (b) R increases for $p < 20$: $R(15) = 1875$ francs

Q3 (a) $h = \dfrac{27}{r^2}$, $S = \pi r^2 + 2\pi rh$ (b) $2\pi r - \dfrac{54\pi}{r^2} = 0$, $r^3 = 27$: $r = 3$ m, $S = 27\pi\ \text{m}^2$

Q4 (a) $(x - 4)^2 + x$ (b) $2x - 7 = 0$, $x = \dfrac{7}{2}$: $AP^2 = \dfrac{15}{4}$, $AP = \dfrac{\sqrt{15}}{2}$

Q5 $P'(n) = -4n + 49 = 0$ at $n = 12.25$, not a whole number; $P(12) = 200$, $P(13) = 199$: 12 chairs, 200 francs

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