Rules of differentiation

Every question on calculus starts here: a derivative found quickly and without a slip. Three families of rules, each with its trap: powers (rewrite first), functions of a function (multiply by the inner derivative), products and quotients (two terms, in the right order). Each part ends with exam-style questions, marked line by line.

I POWERS, SUMS AND MULTIPLES

SL 5.3 · the derivative of x^n, term by term

I.1 The power rule

Property 1 *the power rule*

For every rational n, $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^n) = nx^{n-1}$. A constant has derivative 0; a sum is differentiated term by term; a constant factor stays.

Example 1

Differentiate $y = 2x^5 - 3x^2 + 7x - 4$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Each term: bring the power down, lower it by one	$2 \times 5x^4,\ -3 \times 2x,\ 7,\ 0$
Step 2 Write the derivative	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 10x^4 - 6x + 7$

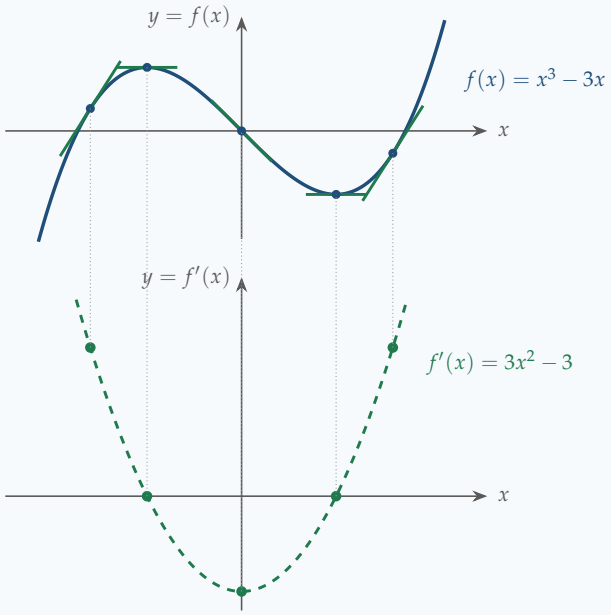


Figure 1 — The gradient of the tangent to $y = x^3 - 3x$, plotted below each x, traces $y = 3x^2 - 3$: the graph of f'.

APPLY

1 $y = x^4 - 5x$

2 $y = 6 - \dfrac{x^3}{3}$

3 $y = 7$

I.2 Rewrite as powers first

Property 2 *rewrite first*

Write every term as a power of x first: $\dfrac{3}{x^2} = 3x^{-2}$, $\sqrt{x} = x^{\frac{1}{2}}$, $\dfrac{2}{\sqrt{x}} = 2x^{-\frac{1}{2}}$, $\dfrac{x^2+5}{x} = x + 5x^{-1}$.

Watch out 1

A quotient is not differentiated top and bottom: divide first, term by term. $\sqrt{x} = x^{\frac{1}{2}}$ has derivative $\dfrac{1}{2\sqrt{x}}$, not $\dfrac{1}{2x}$.

Example 2

Differentiate $y = \dfrac{4}{x^3} + 8\sqrt{x}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Rewrite as powers	$y = 4x^{-3} + 8x^{\frac{1}{2}}$
Step 2 Power rule on each term	$-12x^{-4} + 4x^{-\frac{1}{2}}$
Step 3 Back to fractions if wanted	$-\dfrac{12}{x^4} + \dfrac{4}{\sqrt{x}}$

APPLY

4 $y = \dfrac{1}{x}$

5 $y = 3x^{\frac{2}{3}}$

6 $y = \dfrac{x^4 + 3x}{x}$

I.3 The gradient at a point

Example 3

Let $f(x) = x^3 - 6x$. Find the gradient of the graph at $x = 2$, and the values of x where the gradient is 0.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Differentiate	$f'(x) = 3x^2 - 6$
Step 2 Substitute $x = 2$	$f'(2) = 6$
Step 3 Solve $f'(x) = 0$	$x^2 = 2$: $x = \pm\sqrt{2}$

APPLY

7 $y = x^2 + \dfrac{4}{x}$: the gradient at $x = 2$.

8 $f(x) = x^3 - 12x$: where is $f'(x) = 0$?

II STANDARD DERIVATIVES AND THE CHAIN RULE

SL 5.6 · sin, cos, exponential, logarithm, and a function of a function

II.1 $\sin$, $\cos$, e^x, $\ln x$

Property 3 *the standard derivatives*

$(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = \dfrac{1}{x}$ $(x > 0)$, with x in radians.

Example 4

Differentiate $y = 3\sin x - 2e^x + 5\ln x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Each standard derivative, factor kept	$3\cos x$, $-2e^x$, $\dfrac{5}{x}$
Step 2 Write the derivative	$\dfrac{dy}{dx} = 3\cos x - 2e^x + \dfrac{5}{x}$

APPLY

1 $y = \cos x + x^2$

2 $y = 4\ln x - e^x$

II.2 The chain rule: $(ax+b)^n$, e^{kx}, $\sin kx$

Property 4 *the chain rule*

$y = g(u)$, $u = h(x)$: $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$ — the outer derivative, at the inner function, times the inner derivative. So $((ax+b)^n)' = na(ax+b)^{n-1}$, $(e^{kx})' = ke^{kx}$, $(\sin kx)' = k\cos kx$, $(\ln(ax+b))' = \dfrac{a}{ax+b}$.

Watch out 2

The inner derivative is never forgotten: $(e^{3x})' = 3e^{3x}$. And $(\cos x)' = -\sin x$: the sign is the other mark most often lost.

Example 5

Differentiate $y = (2x+3)^4$ and $y = e^{-5x}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Outer derivative, inner kept	$4(2x+3)^3$ and e^{-5x}
Step 2 Times the inner derivative	$\times 2$ and $\times(-5)$
Step 3 Simplify	$8(2x+3)^3$ and $-5e^{-5x}$

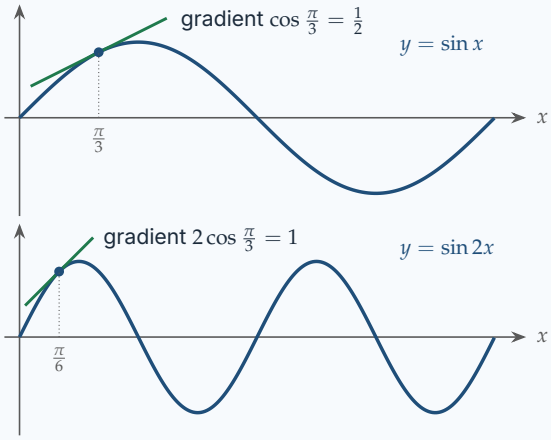


Figure 2 — $y = \sin 2x$ is $\sin x$ squeezed by a factor 2: every slope is twice as steep, the chain rule's factor 2.

APPLY

3 $y = (1 - x)^6$

4 $y = e^{\frac{x}{2}}$

5 $y = \sin 3x$

II.3 The chain rule with any inner function

Example 6

Differentiate $y = \sqrt{x^2 + 5}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Name the inner function	$u = x^2 + 5,\ y = u^{\frac{1}{2}}$
Step 2 $\frac{dy}{du}$ and $\frac{du}{dx}$	$\frac{1}{2}u^{-\frac{1}{2}}$ and $2x$
Step 3 Multiply, put u back	$\frac{x}{\sqrt{x^2 + 5}}$

APPLY

6 $y = \ln(x^2 + 4)$

7 $y = e^{x^3}$

8 $y = (x^3 + 1)^5$

II.4 An exact gradient

Example 7

Let $f(x) = 2\sin 3x$. Find the exact gradient of the graph at $x = \dfrac{\pi}{9}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Chain rule	$f'(x) = 6\cos 3x$
Step 2 Substitute	$6\cos\dfrac{\pi}{3}$
Step 3 Exact value	$6 \times \dfrac{1}{2} = 3$

APPLY

9 $f(x) = \cos 2x$: the gradient at $x = \frac{\pi}{4}$.

10 $f(x) = \mathrm{e}^{2x}$: the gradient at $x = \ln 3$.

III PRODUCTS AND QUOTIENTS

SL 5.6 · the product rule, the quotient rule, and a table of values

III.1 The product rule

Property 5 *the product rule*

$(uv)' = u'v + uv'$: each function differentiated in turn, the other kept.

Example 8

Differentiate $y = x^3\,\mathrm{e}^{2x}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write u, v, u', v'	$u = x^3$, $v = \mathrm{e}^{2x}$, $u' = 3x^2$, $v' = 2\,\mathrm{e}^{2x}$
Step 2 $u'v + uv'$	$3x^2\mathrm{e}^{2x} + 2x^3\mathrm{e}^{2x}$
Step 3 Factorise	$x^2\mathrm{e}^{2x}(3 + 2x)$

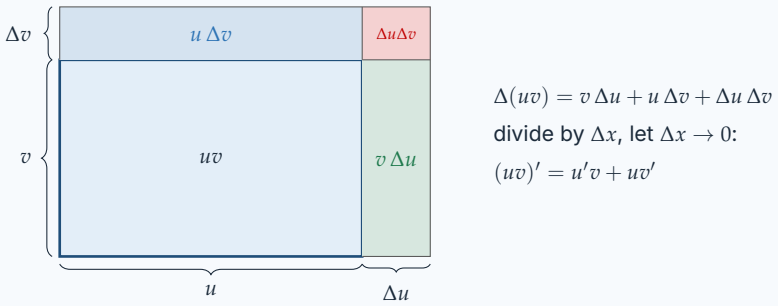


Figure 3 — A rectangle u by v: when x increases a little, the area gains two strips $v\,\Delta u$ and $u\,\Delta v$, and a negligible corner.

APPLY

1 $y = x\sin x$

2 $y = (x+1)\ln x$

III.2 The quotient rule

Property 6 *the quotient rule*

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$: the derivative of the *top* comes first.

Watch out 3

$(uv)'$ is not $u'v'$. In the quotient rule, $uv' - u'v$ gives the opposite sign: every mark after it is lost.

Example 9

Differentiate $y = \dfrac{3x-1}{x+4}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write u, v, u', v'	$u = 3x-1,\ v = x+4,\ u' = 3,\ v' = 1$
Step 2 $\dfrac{u'v - uv'}{v^2}$	$\dfrac{3(x+4)-(3x-1)}{(x+4)^2}$
Step 3 Simplify the numerator	$\dfrac{13}{(x+4)^2}$

APPLY

3 $y = \dfrac{x^2}{x-1}$

4 $y = \dfrac{e^x}{x}$

III.3 Derivatives from a table of values

Example 10

$f(3) = 2$, $f'(3) = 4$, $g(3) = -1$, $g'(3) = 5$. Find $h'(3)$ for $h = fg$, and $k'(3)$ for $k = \dfrac{f}{g}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Product rule at $x = 3$	$h'(3) = 4 \times (-1) + 2 \times 5 = 6$
Step 2 Quotient rule at $x = 3$	$k'(3) = \dfrac{4 \times (-1) - 2 \times 5}{(-1)^2} = -14$

APPLY

5 Same values: $m(x) = \big(f(x)\big)^2$, find $m'(3)$.

6 Same values: $n(x) = 3f(x) - g(x)$, find $n'(3)$.

III.4 Gradients on the GDC

Example 11

Let $f(x) = \ln\left(x^2+1\right) - 0.5x$. Find $f'(3)$, and the values of x where $f'(x) = 0$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Derivative at a point (GDC)	$f'(3) = 0.1$
Step 2 Graph f', or write $\dfrac{2x}{x^2+1} = 0.5$	zeros of f'
Step 3 Read them	$x = 0.268$ or $x = 3.73$

APPLY

7 $f(x) = x \sin x$: $f'(2)$.

IV EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Powers

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = 4x^3 - \frac{6}{x} + 5\sqrt{x}$, for $x > 0$.

(a) Find $f'(x)$. [4]

(b) Find the gradient of the graph of f at the point where $x = 1$. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Rewrite as powers	$-6x^{-1}$ and $5x^{\frac{1}{2}}$ seen *The rewriting earns its own mark: write it down.*	(M1)
Step 2 Power rule on each term	$f'(x) = 12x^2 + 6x^{-2} + \frac{5}{2}x^{-\frac{1}{2}}$ *One A1 for each term. Accept* $\frac{6}{x^2}$ *and* $\frac{5}{2\sqrt{x}}$.	A1A1A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute $x = 1$	$f'(1) = 12 + 6 + \frac{5}{2} = \frac{41}{2}$ *Accept* 20.5. *Follow through from their* (*a*).	A1

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = \dfrac{x^2 - 4x + 9}{x}$, for $x \neq 0$.

(a) Show that $f'(x) = 1 - \dfrac{9}{x^2}$. [3]

(b) Find the values of x for which $f'(x) = 0$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Divide term by term	$f(x) = x - 4 + \dfrac{9}{x} = x - 4 + 9x^{-1}$ *Divide each term of the numerator by x.*	M1
Step 2 Power rule	$f'(x) = 1 - 9x^{-2}$ *One A1 for each term. The constant -4 disappears.*	A1A1
Step 3 Back to the given form	$= 1 - \dfrac{9}{x^2}$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Solve $f'(x) = 0$	$1 - \dfrac{9}{x^2} = 0,\ x^2 = 9$	M1
Step 2 Both values	$x = 3,\ x = -3$ *Both values: $x^2 = 9$ has two solutions.*	A1

EXAM-STYLE QUESTION 3 Paper 1 · no calculator · [Maximum mark: 5]

Find $\frac{dy}{dx}$ for each of the following.

(a) $y = (1-4x)^3$ [2]

(b) $y = e^{x^2+1}$ [2]

(c) $y = \ln(5x+2)$, for $x > -\frac{2}{5}$ [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Outer derivative, inner kept	$3(1-4x)^2 \times (-4)$	(M1)
	The inner derivative is -4: the sign matters.	
Step 2 Times the inner derivative	$= -12(1-4x)^2$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Outer derivative, inner kept	$e^{x^2+1} \times 2x$	(M1)
Step 2 Times the inner derivative	$= 2x\,e^{x^2+1}$	A1
	The exponent stays as it is: e^u has derivative $e^u \times u'$.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\frac{a}{ax+b}$	$\frac{5}{5x+2}$	A1

EXAM-STYLE QUESTION 4

Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = 3\sin 2x + \cos 4x$, where x is in radians.

(a) Find $f'(x)$. [3]

(b) Show that the gradient of the graph of f at $x = \dfrac{\pi}{6}$ is $3 - 2\sqrt{3}$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Chain rule on each term	attempt at the chain rule on either term	(M1)
Step 2 Write the derivative	$f'(x) = 6\cos 2x - 4\sin 4x$	A1A1
	One A1 for each term. $(\cos 4x)' = -4\sin 4x$*: two things to remember, the* 4 *and the minus.*	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute $\dfrac{\pi}{6}$	$f'\left(\dfrac{\pi}{6}\right) = 6\cos\dfrac{\pi}{3} - 4\sin\dfrac{2\pi}{3}$	M1
	Substitute into their $f'(x)$*:* $2 \times \frac{\pi}{6} = \frac{\pi}{3}$ *and* $4 \times \frac{\pi}{6} = \frac{2\pi}{3}$.	
Step 2 Exact values	$= 6 \times \dfrac{1}{2} - 4 \times \dfrac{\sqrt{3}}{2}$	A1
	Exact values on Paper 1: $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$.	
Step 3 Reach the given form	$= 3 - 2\sqrt{3}$	AG

C Products, quotients and tables

EXAM-STYLE QUESTION 5 Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = x^3 \ln x$, for $x > 0$.

(a) Show that $f'(x) = x^2(3\ln x + 1)$. [3]

(b) Find the exact value of x for which $f'(x) = 0$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Product rule	attempt at the product rule	M1
Step 2 $u'v + uv'$	$f'(x) = 3x^2 \ln x + x^3 \times \dfrac{1}{x}$	A1A1
	$u = x^3$, $v = \ln x$: $u'v + uv'$, one A1 for each term.	
Step 3 Factorise	$= 3x^2 \ln x + x^2 = x^2(3\ln x + 1)$	AG
	Factorise x^2: the answer is given, so the last step is shown.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $x^2 \neq 0$: solve $3\ln x + 1 = 0$	$x^2 > 0$, so $3\ln x + 1 = 0,\ \ln x = -\dfrac{1}{3}$	M1
	$x^2 = 0$ is rejected: $x > 0$.	
Step 2 Undo ln	$x = \mathrm{e}^{-\frac{1}{3}}$	A1
	Exact: 0.717 alone earns A0.	

EXAM-STYLE QUESTION 6

Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = \dfrac{2x+1}{x-3}$, for $x \neq 3$.

(a) Show that $f'(x) = -\dfrac{7}{(x-3)^2}$. [3]

(b) Explain why the graph of f has no horizontal tangent. [1]

(c) Find the equation of the tangent to the graph of f at the point where $x = 4$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Quotient rule	$\dfrac{2(x-3)-(2x+1)\times 1}{(x-3)^2}$	M1A1
	M1 for the quotient rule with $u'v$ first; A1 for the correct substitution.	
Step 2 Expand the numerator	$= \dfrac{2x-6-2x-1}{(x-3)^2}$	A1
	Expand the numerator with its brackets: $-(2x+1) = -2x-1$.	
Step 3 Simplify	$= -\dfrac{7}{(x-3)^2}$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The numerator is never 0	$f'(x) \neq 0$ for all x: the numerator is -7	R1
	A horizontal tangent needs $f'(x) = 0$.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Point and gradient at $x = 4$	$f(4) = 9,\ f'(4) = -7$: $y - 9 = -7(x-4)$	M1
Step 2 Write the equation	$y = -7x + 37$	A1

EXAM-STYLE QUESTION 7

Paper 1 · no calculator · [Maximum mark: 7]

The table shows values of the functions f and g and of their derivatives at $x = 1$ and $x = 2$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-2	2	4
2	5	6	1	-3

(a) Let $h(x) = f(x)\,g(x)$. Find $h'(1)$. [2]

(b) Let $k(x) = f(g(x))$. Find $k'(1)$. [3]

(c) Let $m(x) = \dfrac{g(x)}{f(x)}$. Find $m'(2)$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Product rule at $x = 1$	$h'(1) = f'(1)g(1) + f(1)g'(1) = (-2)(2) + (3)(4)$ *The product rule, with the values at $x = 1$ read from the table.*	M1
Step 2 Compute	$= 8$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Chain rule: f' at $g(1)$	$k'(1) = f'(g(1)) \times g'(1)$ *The chain rule: f' is taken at $g(1)$, not at 1.*	M1
Step 2 Read $f'(2)$	$= f'(2) \times 4$ *$g(1) = 2$: so it is $f'(2) = 6$ that is needed.*	A1
Step 3 Compute	$= 6 \times 4 = 24$ *$f'(1) \times g'(1) = -8$ is the classic error.*	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Quotient rule at $x = 2$	$m'(2) = \dfrac{g'(2)f(2) - g(2)f'(2)}{(f(2))^2} = \dfrac{(-3)(5) - (1)(6)}{25}$	M1
Step 2 Compute	$= -\dfrac{21}{25}$ *Accept -0.84.*	A1

D With the GDC (Paper 2)

EXAM-STYLE QUESTION 8 Paper 2 · calculator · [Maximum mark: 6]

Let $f(x) = (x^2 + 1)\,e^{-0.4x}$, for $0 \leq x \leq 8$.

(a) Find the gradient of the graph of f at the point where $x = 2$. [2]

(b) Find the values of x for which $f'(x) = 0.5$. [2]

(c) Find the greatest value of $f'(x)$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Derivative at $x = 2$ on the GDC	use of GDC, or $f'(x) = 2x\,e^{-0.4x} - 0.4\,(x^2 + 1)\,e^{-0.4x}$	(M1)
	$f'(2) = 0.899$ $(0.898657\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Graph f' and $y = 0.5$: intersections	$x = 0.584$ $(0.583993\ldots)$	A1
	A sketch of f' and $y = 0.5$ shows two intersections.	
	$x = 3.28$ $(3.27988\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Maximum of the graph of f'	the maximum of the graph of f', at $x = 1.61$	(M1)
	The greatest gradient: the maximum of f', not of f.	
Step 2 Read the value	0.936 $(0.936493\ldots)$	A1

EXAM-STYLE QUESTION 9

Paper 2 · calculator · [Maximum mark: 5]

Let $f(x) = \sin\left(x^2\right)$, for $0 \leq x \leq 2.5$, where x is in radians.

(a) Find $f'(x)$. [2]

(b) Find the values of x for which the gradient of the graph of f is 1. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Chain rule	$f'(x) = 2x\cos\left(x^2\right)$	A1A1
	A1 for $\cos\left(x^2\right)$, A1 for the factor $2x$.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Graph f' and $y = 1$: intersections	$x = 0.519 \;\; (0.518652\ldots)$	A1
	Three intersections of $y = 2x\cos(x^2)$ with $y = 1$: a sketch on the GDC shows them all.	
	$x = 1.03 \;\; (1.03199\ldots)$	A1
	$x = 2.22 \;\; (2.22245\ldots)$	A1
	Radians: in degrees the graph is a different curve.	

V WHICH METHOD? READ THE QUESTION

5.6 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
$\frac{1}{x^k}$, $\sqrt{x}$, *or a fraction with one term below*	powers	rewrite as x^{-k}, $x^{\frac{1}{2}}$, or divide term by term; then nx^{n-1} P2
$(ax+b)^n$, $e^{\cdots}$, $\sin(\ldots)$, $\ln(\ldots)$	a function of a function	the outer derivative, at the inner function, times the inner derivative P4
two functions multiplied	u, v	$u'v + uv'$: write u, v, u', v' first P5
x *below the line, more than one term*	u over v	$\frac{u'v - uv'}{v^2}$, top derivative first P6
"show that $f'(x) = \ldots$*"*	the target	every step; factorise at the end to reach the given form
"exact value", "show that the gradient is $3 - 2\sqrt{3}$*"*	Paper 1	exact values: $\frac{\pi}{6}$, $\sqrt{3}$, e, $\ln 2$; no decimals
a table of f, g, f', g'	values only no formula	the rule at the point; for $f(g(x))$, f' at $g(a)$, not at a P4
"where is $f'(x) = k$*?"*	an equation	solve $f'(x) = k$; every solution in the domain
Paper 2, "find $f'(a)$*", or* $f'(x) = k$ *too hard to solve*	the GDC is allowed	Part V: the derivative at a point, the graph of f', an intersection

In the exam 1 *command terms*

Find $f'(x)$: any correct form, simplified or not, unless a form is asked. **Show that**: every step, the last line is the answer given. **Hence**: use the previous part. **Exact**: no decimal, no GDC value. **Write down**: no working needed.

VI ON THE GDC: PAPER 2

SL 5.3 · 5.6 · NumWorks · every calculator move of this topic

1 The derivative at a point

Calculation → Toolbox → Calculus → Derivative

A value of $f'(a)$, with no rule at all. **On paper:** "$f'(2) = 0.899$ (GDC)". It also checks a derivative found by hand: evaluate yours at the same point.

CALCULATION

```
diff((x^2+1)e^(-0.4x),x,2)
0.8986579282
```

2 Check a derivative found by hand

Grapher → g(x) = derivative of f → h(x) = yours

Plot the GDC's f' and your own expression as a second function: if the graphs do not coincide, a rule went wrong. Thirty seconds that save a whole question.

GRAPH

```
f'(x) plotted by the GDC
g(x)=2(-x^2+5x-1)e^(-0.4x)/5
the two curves coincide
```

3 Where is $f'(x) = k$?

Grapher → g(x) = derivative of f → Find → X given Y

Plot f' and the line $y = k$, then Find → X given Y, and the right arrow for the next one. **On paper:** the equation $f'(x) = k$, then every solution in the domain.

GRAPH · INTERSECTION

```
f'(x) plotted, g(x)=1
x=0.5186522 y=1
x=1.031990 y=1
x=2.222455 y=1
```

4 The greatest gradient

Grapher → g(x) = derivative of f → Find → Maximum

The steepest point of f is the maximum of the graph of f'. **On paper:** "the maximum of f'", then its value.

GRAPH · MAXIMUM

```
f'(x) plotted
x=1.608835 y=0.9364937
```

ANSWERS TO THE APPLY QUESTIONS

I.1 $4x^3 - 5$

I.2 $-x^2$

I.3 0

I.4 $-\dfrac{1}{x^2}$

I.5 $2x^{-\frac{1}{3}}$

I.6 $3x^2$

I.7 3

I.8 $x = \pm 2$

II.1 $-\sin x + 2x$

II.2 $\dfrac{4}{x} - \mathrm{e}^x$

II.3 $-6(1-x)^5$

II.4 $\frac{1}{2}\mathrm{e}^{\frac{x}{2}}$

II.5 $3\cos 3x$

II.6 $\dfrac{2x}{x^2+4}$

II.7 $3x^2\mathrm{e}^{x^3}$

II.8 $15x^2\left(x^3+1\right)^4$

II.9 -2

II.10 18

III.1 $\sin x + x\cos x$

III.2 $\ln x + \dfrac{x+1}{x}$

III.3 $\dfrac{x^2-2x}{(x-1)^2}$

III.4 $\dfrac{\mathrm{e}^x(x-1)}{x^2}$

III.5 $2 \times 2 \times 4 = 16$

III.6 7

III.7 0.0770

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