Rules of differentiation

Skills sheet · SL AA 5.3 · 5.6

KEY FACTS

$(x^n)' = nx^{n-1}$, any rational n P1

$(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(e^x)' = e^x$, $(\ln x)' = \frac{1}{x}$ P3

Chain: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ P4

$(uv)' = u'v + uv'$, $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ P5 P6

Rewrite as powers before anything else P2.
The inner derivative is never forgotten W2.
$uv' - u'v$ gives the opposite sign W3.

WHAT UNLOCKS

- $\frac{3}{x^2}, \sqrt{x}, \frac{1}{\sqrt{x}} \rightarrow 3x^{-2}, x^{\frac{1}{2}}, x^{-\frac{1}{2}}$.
- something inside a bracket, an exponent, a sine or a logarithm $\rightarrow$ the chain rule: multiply by its derivative.
- "show that $f'(x) = \ldots$" with a factorised answer $\rightarrow$ take out the common factor of the two terms.
- a table of f, g, f', g' $\rightarrow$ write the rule first, then read each value at the right x.

WORKED EXAMPLE ①

Find $\frac{dy}{dx}$ when $y = (2x^2 - 1)^4$. no GDC

outer: $4u^3$, inner: $u = 2x^2 - 1$, $u' = 4x$ (M1)

$\frac{dy}{dx} = 4(2x^2 - 1)^3 \times 4x$ A1

$= 16x(2x^2 - 1)^3$ A1

Method

How do I use the chain rule?

① Name the inner function u.
② Differentiate the outer one, u untouched.
③ Multiply by u'.
④ Tidy: constants in front.

Forgetting $\times u'$: A0, every time.

WORKED EXAMPLE ②

Show that the derivative of $y = \frac{x}{x+2}$ is $\frac{2}{(x+2)^2}$. no GDC

$u = x, u' = 1, v = x + 2, v' = 1$ (M1)

$\frac{1 \times (x+2) - x \times 1}{(x+2)^2}$ A1

$= \frac{2}{(x+2)^2}$ AG

Method

How do I use the quotient rule?

① Write u, v, u', v' in a column.
② $u'v$ first, then $-uv'$, over v^2.
③ Expand the numerator with brackets.
④ Leave the denominator squared.

FLUENCY

●○○ **1.** $y = 5x^4 - \frac{3}{x} + 7$ no GDC

●○○ **2.** $y = 6\sqrt{x} - \frac{2}{x^3}$ no GDC

●○○ **3.** $y = \dfrac{x^3 - 2x}{x^2}$, $x \neq 0$ no GDC

●○○ **4.** $y = (3x + 2)^5$ no GDC

●○○ **5.** $y = x^2 \, \mathrm{e}^{3x}$ no GDC

●○○ **6.** $y = \ln\left(x^2 + 1\right)$ no GDC

REASONING

●●○ **Question 1.** Let $f(x) = 2\cos 3x - \sin \dfrac{x}{2}$, with x in radians. [no GDC]

(a) Find $f'(x)$. [3]

(b) Find the exact value of $f'\left(\dfrac{\pi}{3}\right)$. [2]

●●○ **Question 2.** Let $f(x) = \dfrac{x}{x^2 + 1}$. [no GDC]

(a) Show that $f'(x) = \dfrac{1 - x^2}{(x^2 + 1)^2}$. [3]

(b) Find the coordinates of the points at which the tangent is horizontal. [3]

●●○ **Question 3.** Let $g(x) = \mathrm{e}^{-x} \sin x$, with x in radians. [no GDC]

(a) Find $g'(x)$. [3]

(b) Show that the tangent to the graph of g at $x = \dfrac{\pi}{4}$ is horizontal. [2]

PROBLEMS

●●● **Question 4.** The functions f and g satisfy $f(0) = 1$, $f'(0) = 3$, $g(0) = 2$, $g'(0) = -1$, $f(2) = 4$, $f'(2) = 5$. [no GDC]

(a) Find the derivative of $f(x)\,g(x)$ at $x = 0$. [2]

(b) Find the derivative of $f\big(g(x)\big)$ at $x = 0$. [2]

(c) Find the derivative of $\dfrac{f(x)}{g(x)}$ at $x = 0$. [2]

●●● **Question 5.** Let $f(x) = x\sqrt{4 - x}$, for $0 \leq x \leq 4$. [no GDC]

(a) Show that $f'(x) = \dfrac{8 - 3x}{2\sqrt{4 - x}}$. [4]

(b) Find the exact coordinates of the point at which the tangent is horizontal. [3]

ANSWERS

1 $20x^3 + 3x^{-2}$

2 $3x^{-\frac{1}{2}} + 6x^{-4}$, that is $\dfrac{3}{\sqrt{x}} + \dfrac{6}{x^4}$

3 $y = x - 2x^{-1}$: $1 + \dfrac{2}{x^2}$

4 $15(3x+2)^4$

5 $2x\,\mathrm{e}^{3x} + 3x^2\,\mathrm{e}^{3x} = x\,\mathrm{e}^{3x}(2+3x)$

6 $\dfrac{2x}{x^2+1}$

Q1 (a) $f'(x) = -6\sin 3x - \dfrac{1}{2}\cos\dfrac{x}{2}$ (b) $-6\sin\pi - \dfrac{1}{2}\cos\dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{4}$

Q2 (a) $\dfrac{(x^2+1) - x(2x)}{(x^2+1)^2}$ (b) $1 - x^2 = 0$: $\left(1,\ \frac{1}{2}\right)$ and $\left(-1,\ -\frac{1}{2}\right)$

Q3 (a) $g'(x) = -\mathrm{e}^{-x}\sin x + \mathrm{e}^{-x}\cos x = \mathrm{e}^{-x}(\cos x - \sin x)$ (b) $\cos\frac{\pi}{4} = \sin\frac{\pi}{4}$, so $g'\left(\frac{\pi}{4}\right) = 0$

Q4 (a) $f'(0)g(0) + f(0)g'(0) = 6 - 1 = 5$ (b) $f'\big(g(0)\big)\,g'(0) = f'(2) \times (-1) = -5$ (c) $\dfrac{3\times 2 - 1\times(-1)}{2^2} = \dfrac{7}{4}$

Q5 (a) $\sqrt{4-x} - \dfrac{x}{2\sqrt{4-x}} = \dfrac{2(4-x)-x}{2\sqrt{4-x}}$ (b) $x = \dfrac{8}{3}$: $\left(\dfrac{8}{3},\ \dfrac{16\sqrt{3}}{9}\right)$

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