Marked example 3

Paper 1 style · No calculator

3 15 marks SL 5.3 · 5.4 · 5.8 ●●○ Typical to demanding

Let $f(x) = x^3 + ax^2 + bx$, where a and b are constants. The graph of f has a stationary point at $(1, -5)$.

(a) Show that $2a + b = -3$ and $a + b = -6$. **[3]**

(b) Hence find the value of a and the value of b. **[2]**

(c) Find the coordinates of the other stationary point of the graph of f, and determine the nature of both stationary points. **[4]**

(d) Find the equation of the tangent to the graph of f at the point where $x = -1$. **[3]**

(e) Show that this tangent meets the graph of f at one point only. **[3]**

MARKSCHEME (a)

$f'(x) = 3x^2 + 2ax + b$ **A1**

$f'(1) = 0$: $3 + 2a + b = 0$, so $2a + b = -3$ **A1 AG**

$f(1) = -5$: $1 + a + b = -5$, so $a + b = -6$ **A1 AG**

Total 3 marks

MARKSCHEME (b)

attempt to solve the two equations simultaneously **(M1)**

$a = 3,\ b = -9$ **A1**

Total 2 marks

MARKSCHEME (c)

$f'(x) = 3x^2 + 6x - 9 = 3(x + 3)(x - 1)$ **(M1)**

other stationary point $(-3, 27)$ **A1**

$f''(x) = 6x + 6$: $f''(-3) = -12 < 0$, so $(-3, 27)$ is a local maximum **R1**

$f''(1) = 12 > 0$, so $(1, -5)$ is a local minimum **R1**

Total 4 marks

Note Accept a correct sign diagram of f' for each R1. FT from their values of a and b.

MARKSCHEME (d)

$f(-1) = 11$ **A1**

$f'(-1) = 3 - 6 - 9 = -12$ **A1**

$y = -12x - 1$ **A1**

Total 3 marks

MARKSCHEME (e)

$x^3 + 3x^2 - 9x = -12x - 1$ **M1**

$x^3 + 3x^2 + 3x + 1 = 0$ **A1**

$(x+1)^3 = 0$, so $x = -1$ is the only solution: one point only **R1**

Total 3 marks

Note Accept a correct factorisation $(x+1)(x^2+2x+1)$ followed by a correct conclusion. The point $(-1, 11)$ is the point of inflexion of the graph, but this is not required.

Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization. © 2026 Hence