Exercise 2

Paper 1 style · No calculator

2 7 marks SL 5.8 ●●○ Typical

Let $f(x) = x^3 - 3x^2 + 3x + 4$.

(a) Show that the graph of f has exactly one stationary point, at $(1, 5)$. **[3]**

(b) Show that $f''(1) = 0$, and explain why the second derivative test gives no conclusion at $x = 1$. **[2]**

(c) Show that $(1, 5)$ is a point of inflexion of the graph of f. **[2]**

The markscheme is on the next page: try the question first. Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization.
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MARKSCHEME (a)

$f'(x) = 3x^2 - 6x + 3$ **A1**

$= 3(x-1)^2 = 0$ only for $x = 1$ **M1**

$f(1) = 1 - 3 + 3 + 4 = 5$ **A1**

Total 3 marks

MARKSCHEME (b)

$f''(x) = 6x - 6,\ f''(1) = 0$ **A1**

the test needs $f''(1) \neq 0$: with $f''(1) = 0$ the point could be a maximum, a minimum or an inflexion **R1**

Total 2 marks

MARKSCHEME (c)

$f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$ **M1**

f'' changes sign at $x = 1$, so $(1, 5)$ is a point of inflexion **R1**

Total 2 marks

Note It is a stationary point of inflexion: $f'(x) = 3(x-1)^2 \geq 0$ on both sides.