Stationary points and inflexion

Skills sheet · SL AA 5.8

KEY FACTS

Stationary point: $f'(a) = 0$, then $(a, f(a))$ D1

f': + to − max, − to + min, no change: inflexion P1

$f''(a) < 0$ max, $f''(a) > 0$ min, $f''(a) = 0$: no conclusion P2

Given (a, b): $f'(a) = 0$ **and** $f(a) = b$ P4

"Coordinates": x and y W1.

$f''(a) = 0$: go back to the sign of f' W2.

On an interval, the ends are candidates too.

WHAT UNLOCKS

- "stationary", "turning point", "horizontal tangent" $\rightarrow$ $f'(x) = 0$.
- "justify", "nature" $\rightarrow$ a test, its numbers, then the word: R1 before A1.
- two unknown constants and a stationary point $\rightarrow$ two equations, $f'(a) = 0$ and $f(a) = b$.
- "$f(x) = k$ has three solutions" $\rightarrow$ the line $y = k$ strictly between the minimum and maximum values.

WORKED EXAMPLE ①

Find the stationary points of $f(x) = x^3 - 3x^2 + 4$ and their nature. no GDC

$f'(x) = 3x^2 - 6x = 3x(x-2) = 0$ — M1

$(0, 4)$ and $(2, 0)$ — A1A1

$f''(x) = 6x - 6$: $f''(0) = -6 < 0$, $f''(2) = 6 > 0$ — R1

maximum $(0, 4)$, minimum $(2, 0)$ — A1

Method

How do I classify a stationary point?

① Solve $f'(x) = 0$; then $y = f(x)$.
② $f''(a)$: its sign gives the answer.
③ If $f''(a) = 0$: signs of f' either side.
④ Write the reason, then the word.

"Maximum" with no reason: R0A0.

WORKED EXAMPLE ②

$f(x) = x^2 + bx + c$ has a minimum point at $(3, -1)$. Find b and c. no GDC

$f'(3) = 6 + b = 0$: $b = -6$ — M1A1

$f(3) = 9 - 18 + c = -1$: $c = 8$ — M1A1

Method

How do I find unknown coefficients?

① Stationary at $x = a$: $f'(a) = 0$.
② On the graph: $f(a) = b$.
③ Solve the two equations together.
④ Check the nature if it is given.

FLUENCY

●○○ **1.** Stationary point of $y = x^2 - 8x + 3$ and its nature. no GDC

●○○ **2.** Stationary points of $y = x^3 - 27x$. no GDC

●○○ **3.** Stationary point of $y = x + \frac{9}{x}$, $x > 0$. no GDC

●○○ **4.** $f(x) = 2x^3 - 3x^2 - 12x$: stationary points, second derivative test. no GDC

●○○ **5.** Stationary point of $y = x\,e^x$. no GDC

●○○ **6.** $y = x^4$: the nature of $(0, 0)$. no GDC

REASONING

●●● **Question 1.** Let $f(x) = x^3 - 6x^2 + 12x$. [no GDC]

(a) Show that $f'(x) = 3(x-2)^2$. [2]

(b) Find the stationary point and show that it is a point of inflexion. [4]

●●● **Question 2.** Let $f(x) = x^2 + \dfrac{16}{x}$, for $x \neq 0$. [no GDC]

(a) Find the coordinates of the stationary point. [3]

(b) Use the second derivative to determine its nature. [2]

●●● **Question 3.** The graph of $f(x) = ax^3 + bx$ has a stationary point at $(1, -4)$. [no GDC]

Find the value of a and the value of b. [4]

PROBLEMS

●●● **Question 4.** Let $f(x) = x^3 - 3x^2 - 9x + k$, where k is a constant. [no GDC]

(a) Find the x-coordinates of the stationary points of f. [3]

(b) Find the values of k for which the equation $f(x) = 0$ has three solutions. [4]

●●● **Question 5.** Let $f(x) = x^3 - 3x$, for $-2 \leq x \leq 3$. [no GDC]

Find the greatest and the least values of $f(x)$ on this interval. Justify your answer. [5]

ANSWERS

1 $2x - 8 = 0$: $(4,\ -13)$, minimum

2 $3x^2 - 27 = 0$: $(3,\ -54)$ minimum, $(-3,\ 54)$ maximum

3 $1 - \dfrac{9}{x^2} = 0$: $(3,\ 6)$, minimum

4 $6(x+1)(x-2) = 0$; $f'' = 12x - 6$: $(-1,\ 7)$ maximum, $(2,\ -20)$ minimum

5 $(x+1)\mathrm{e}^x = 0$: $\left(-1,\ -\mathrm{e}^{-1}\right)$, minimum

6 $f''(0) = 0$, no conclusion; $f' = 4x^3$ changes from $-$ to $+$: minimum

Q1 (a) $3x^2 - 12x + 12 = 3(x-2)^2$ (b) $(2,\ 8)$; $f''(x) = 6x - 12$ changes sign at $x = 2$

Q2 (a) $2x - \dfrac{16}{x^2} = 0$, $x^3 = 8$: $(2,\ 12)$ (b) $f''(x) = 2 + \dfrac{32}{x^3}$, $f''(2) = 6 > 0$: minimum

Q3 $3a + b = 0$ and $a + b = -4$: $a = 2$, $b = -6$

Q4 (a) $3(x+1)(x-3) = 0$: $x = -1$, $x = 3$ (b) maximum value $f(-1) = 5 + k$, minimum value $f(3) = k - 27$; three solutions when $k - 27 < 0 < 5 + k$: $-5 < k < 27$

Q5 stationary points $f(-1) = 2$, $f(1) = -2$; ends $f(-2) = -2$, $f(3) = 18$: greatest value 18 (at $x = 3$), least value -2 (at $x = 1$ and $x = -2$)

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