Tangents and normals

The gradient of a curve at a point is $f'(a)$. Every question on this topic turns that number into the equation of a line, the *tangent* or the *normal*, or works backwards from a gradient to a point. Each part ends with exam-style questions, marked line by line.

I THE TANGENT AT A POINT

SL 5.4 · tangents at a given point, and their equations

I.1 The tangent at $x = a$

Definition 1 *tangent*

The **tangent** to the graph of f at $P\big(a,\ f(a)\big)$ is the line through P whose gradient is the gradient of the curve at P, $f'(a)$.

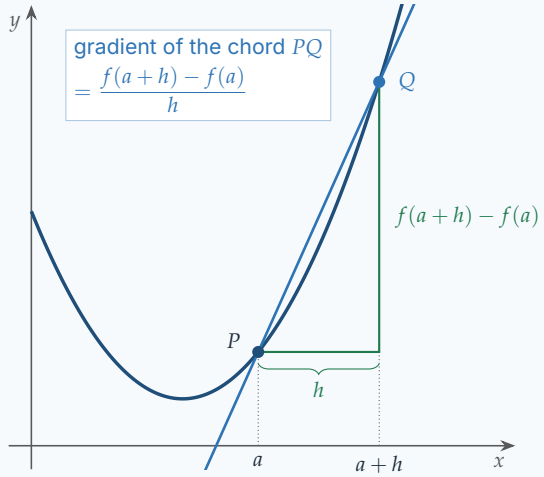


Figure 1 — P at $x = a$ and Q at $x = a + h$: the gradient of the chord is "rise over run".

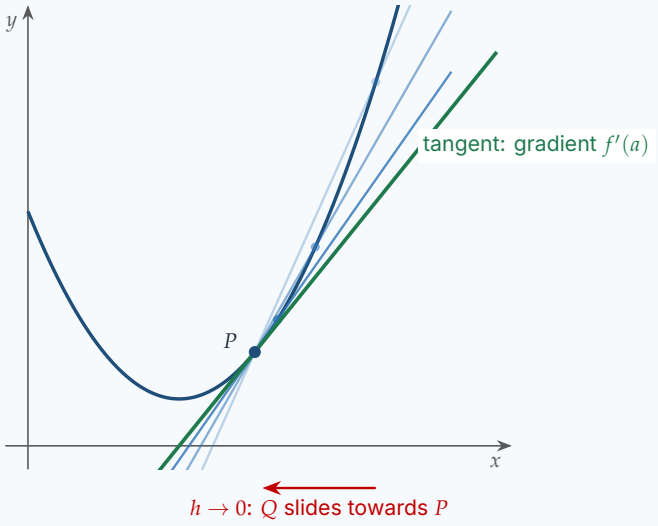


Figure 2 — As h shrinks, the chord turns towards one line: the tangent, gradient $f'(a)$.

Property 1 *equation of the tangent*

The tangent at $x = a$: $y - f(a) = f'(a)\,(x - a)$.

Example 1

Let $f(x) = x^2 - 3x + 4$. Find the equation of the tangent to the graph of f at $x = 3$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The point: $f(a)$	$f(3) = 4$: $(3, 4)$
Step 2 The gradient: $f'(a)$	$f'(x) = 2x - 3$, $f'(3) = 3$
Step 3 Point-gradient form	$y - 4 = 3(x - 3)$
Step 4 Simplify	$y = 3x - 5$

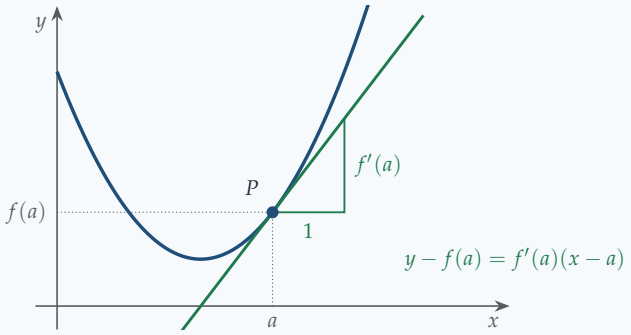


Figure 3 — The three numbers of the equation on the picture: the point $(a,\ f(a))$, and one step across, $f'(a)$ steps up — the steps of Example 1.

APPLY

1 $y = x^3 - 2x$ at $x = 2$.

2 $y = \dfrac{6}{x}$ at $x = 3$.

I.2 With e^x, $\ln x$, $\sin x$: exact values

Example 2

Find the equation of the tangent to $y = 3\,e^x$ at $x = 0$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The point	$3\,e^0 = 3$: $(0, 3)$
Step 2 The gradient	$y' = 3\,e^x$, 3 at $x = 0$
Step 3 The line	$y = 3x + 3$

APPLY

3 $y = \ln x$ at $x = 1$.

4 $y = \sin x$ at $x = \pi$.

I.3 Where a tangent meets the axes

Example 3

The tangent to $y = x^2 + 1$ at $x = 2$ meets the axes at A and B. Find A, B and the area of the triangle OAB.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The tangent	$(2, 5)$, gradient 4: $y = 4x - 3$
Step 2 $y = 0$ and $x = 0$ in the line	$A\left(\frac{3}{4}, 0\right)$, $B(0, -3)$
Step 3 Half base times height	$\frac{1}{2} \times \frac{3}{4} \times 3 = \frac{9}{8}$

APPLY

5 The tangent $y = -3x + 6$: where it meets the axes, and the area of the triangle with them.

II THE NORMAL AT A POINT

SL 5.4 · normals at a given point, and their equations

II.1 The normal

Definition 2 *normal*

The **normal** at P is the line through P **perpendicular** to the tangent at P.

Property 2 *perpendicular gradients*

Perpendicular lines: $m_1 m_2 = -1$. When $f'(a) \neq 0$, the normal at $x = a$ has gradient $-\dfrac{1}{f'(a)}$: $y - f(a) = -\dfrac{1}{f'(a)}(x - a)$.

Watch out 1

The normal uses $-\dfrac{1}{f'(a)}$: not $f'(a)$, not $-f'(a)$, not $\dfrac{1}{f'(a)}$.

Example 4

Let $f(x) = x^2 - 4x + 1$. Find the equation of the normal to the graph of f at $x = 3$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Point and gradient of the tangent	$f(3) = -2,\ f'(3) = 2$
Step 2 Normal gradient: $-\dfrac{1}{f'(a)}$, first	$-\dfrac{1}{2}$
Step 3 The line through the point	$y + 2 = -\dfrac{1}{2}(x - 3),\ y = -\dfrac{1}{2}x - \dfrac{1}{2}$

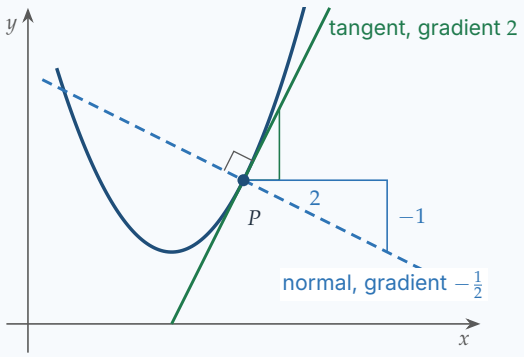


Figure 4 — The gradient triangle of the tangent turned through a right angle: "one across, two up" becomes "two across, one down", as in Example 4.

APPLY

1 The normal to $y = x^3$ at $x = -1$.

2 The normal to $y = 2\sqrt{x}$ at $x = 4$.

II.2 Horizontal tangents, vertical normals

Note 1 *when* $f'(a) = 0$

The tangent is horizontal, $y = f(a)$; the normal is vertical, $x = a$: it has no gradient and no equation $y = mx + c$.

Example 5

Let $f(x) = x^3 - 3x$. Find the points where the tangent is horizontal, and the equations of the normals there.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Solve $f'(x) = 0$	$3x^2 - 3 = 0$: $x = \pm 1$
Step 2 The points	$(1, -2)$ and $(-1, 2)$
Step 3 Vertical normals	$x = 1$ and $x = -1$

APPLY

3 $f(x) = x^2 - 6x + 5$: the horizontal tangent and the normal there.

II.3 A normal that meets the curve again

Example 6

The normal to $y = x^2$ at $P(1, 1)$ meets the curve again at Q. Find Q.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The normal	gradient $-\frac{1}{2}$: $y = -\frac{1}{2}x + \frac{3}{2}$
Step 2 Curve = line	$2x^2 + x - 3 = 0$
Step 3 One root is P: factorise	$(x - 1)(2x + 3) = 0$
Step 4 Keep the other root	$Q\left(-\frac{3}{2}, \frac{9}{4}\right)$

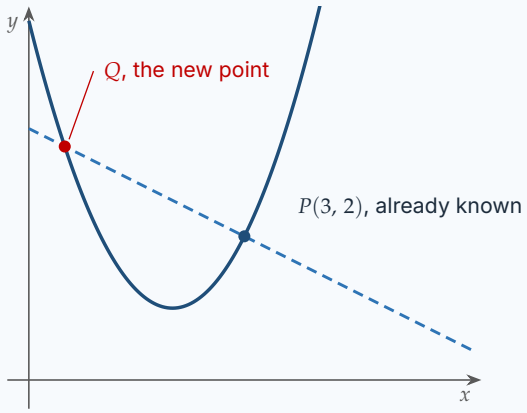


Figure 5 — A line meets a parabola at most twice: one root of the equation is the point already known, the other is the answer.

APPLY

4 The normal to $y = x^2 - 4x$ at $(0, 0)$ meets the curve again at Q.

III WORKING BACK FROM A GRADIENT

SL 5.4 · SL 5.6

III.1 A tangent parallel to a line

Property 3 *a given gradient*

The tangent has gradient m where $f'(x) = m$: **parallel** to a line of gradient m when $f'(x) = m$, **perpendicular** to it when $f'(x) = -\dfrac{1}{m}$.

Watch out 2

Solve $f'(x) = m$, not $f(x) = mx + c$ (that gives where the curve crosses the line). The y-coordinate comes from f, not from the line.

Example 7

Find the point of $y = x^2 + 3x$ where the tangent is parallel to $y = 7x - 2$, and that tangent.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Parallel: $f'(x) = 7$	$2x + 3 = 7,\ x = 2$
Step 2 y from the curve	$y = 10$: $(2, 10)$
Step 3 The tangent	$y = 7x - 4$

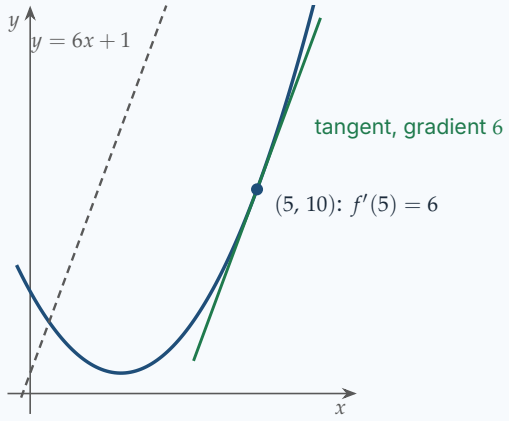


Figure 6 — The given line and the tangent have the same gradient; the point comes from $f'(x) = m$.

APPLY

1 $y = x^3 + 1$: the points where the gradient is 12.

2 $y = 4\sqrt{x}$: the point where the tangent is parallel to $y = x + 5$.

III.2 A tangent perpendicular to a line

Example 8

Find the point of $y = x^2 - x$ where the tangent is perpendicular to $x + 5y = 10$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Gradient of the line	$y = -\frac{1}{5}x + 2$: $-\frac{1}{5}$
Step 2 Perpendicular: $f'(x) = 5$	$2x - 1 = 5,\ x = 3$
Step 3 y from the curve	$(3, 6)$

APPLY

3 $y = 2x^2$: the point where the tangent is perpendicular to $x + 4y = 3$.

III.3 A line that touches the curve

Property 4 *a line that touches the curve*

$y = mx + c$ is a tangent at $x = t$ when **both**: $f(t) = mt + c$ (same point) and $f'(t) = m$ (same gradient).

Example 9

The line $y = 3x + k$ is a tangent to $y = x^2 + x + 5$. Find k.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Same gradient	$2x + 1 = 3,\ x = 1$
Step 2 Same point at $x = 1$	$7 = 3 + k$
Step 3 Solve	$k = 4$

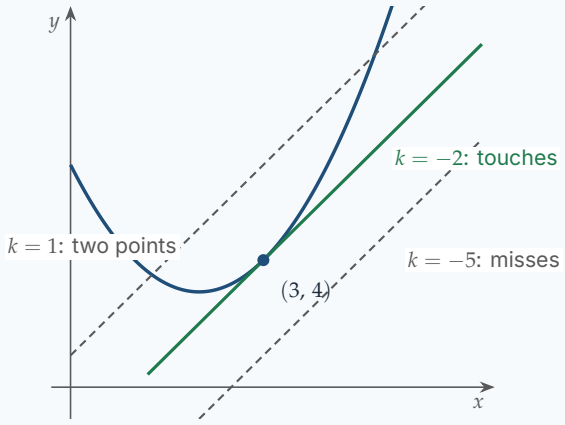


Figure 7 — Three parallel lines $y = 2x + k$ and $y = x^2 - 4x + 7$: only one k gives one common point with a common gradient.

APPLY

4 $y = -2x + k$ is a tangent to $y = x^2 + 4$. Find k.

5 The tangent to $y = ax^2 + b$ at $x = 1$ is $y = 4x - 1$. Find a and b.

III.4 Tangents on the GDC

Example 10

Let $f(x) = \ln x + x^2$. Find the equation of the tangent at $x = 2$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The gradient at $x = 2$ (GDC)	4.5
Step 2 The point, stored	$f(2) = 4.693147\ldots$
Step 3 The line, 3 s.f.	$y = 4.5x - 4.31$

APPLY

6 $f(x) = e^{0.5x}$: the tangent at $x = 1$.

IV EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Tangents

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = x^3 - 4x + 1$.

(a) Find $f'(x)$. [2]

(b) Find the equation of the tangent to the graph of f at the point where $x = 2$. Give your answer in the form $y = mx + c$. [4]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Differentiate	$f'(x) = 3x^2 - 4$	A1A1
	One A1 for each term. A constant left in the derivative, $3x^2 - 4 + 1$, loses the second A1.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The point: $f(2)$	$f(2) = 8 - 8 + 1 = 1$, so the point is $(2, 1)$	(A1)
	The point must be on the curve: *find $f(2)$ yourself, it is not given.*	
Step 2 The gradient: $f'(2)$	$f'(2) = 3(4) - 4 = 8$	(A1)
Step 3 Point-gradient form	$y - 1 = 8(x - 2)$	M1
	The M1 is for substituting your *point and* your *gradient into a line.*	
Step 4 Simplify	$y = 8x - 15$	A1
	"In the form $y = mx + c$": $y - 1 = 8(x - 2)$ alone does not earn this mark.	

EXAM-STYLE QUESTION 2

Paper 1 · no calculator · [Maximum mark: 7]

Let $f(x) = \dfrac{8}{x}$, for $x > 0$.

(a) Find the equation of the tangent to the graph of f at the point where $x = 2$. [4]

The tangent meets the x-axis at A and the y-axis at B. Let O be the origin.

(b) Find the coordinates of A and of B. [2]

(c) Find the area of the triangle OAB. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Rewrite and differentiate	$f(x) = 8x^{-1},\ f'(x) = -8x^{-2}$	(A1)
	Rewrite $\dfrac{8}{x}$ as a power first: the derivative of a quotient is not needed.	
Step 2 Point and gradient at $x = 2$	$f(2) = 4,\ f'(2) = -2$	(A1)
Step 3 Point-gradient form	$y - 4 = -2(x - 2)$	M1
Step 4 Simplify	$y = -2x + 8$	A1
	No form is asked: $2x + y = 8$ is also correct.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $y = 0$ in the line	$A(4,\ 0)$	A1
	Put $y = 0$ in the tangent, not in f.	
Step 2 $x = 0$ in the line	$B(0,\ 8)$	A1
	Follow through from their tangent.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Half base times height	$\text{area} = \dfrac{1}{2} \times 4 \times 8 = 16$	A1

EXAM-STYLE QUESTION 3

Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = x \ln x$, for $x > 0$.

(a) Show that $f'(x) = \ln x + 1$. [2]

(b) Find the equation of the tangent to the graph of f at the point where $x = \mathrm{e}$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Product rule	product rule: $f'(x) = 1 \times \ln x + x \times \dfrac{1}{x}$	M1A1
	"Show that": every step is written, and the last line is the answer given.	
	$f'(x) = \ln x + 1$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Point and gradient at $x = \mathrm{e}$	$f(\mathrm{e}) = \mathrm{e}, \ f'(\mathrm{e}) = \ln \mathrm{e} + 1 = 2$	(A1)
	$\ln \mathrm{e} = 1$*: exact values only on Paper 1.*	
Step 2 Point-gradient form	$y - \mathrm{e} = 2(x - \mathrm{e})$	M1
Step 3 Simplify	$y = 2x - \mathrm{e}$	A1

B Normals

EXAM-STYLE QUESTION 4 Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = x^2 - 5x + 4$. The point $P(3, -2)$ lies on the graph of f.

(a) Find the gradient of the graph of f at P. [2]

(b) Find the equation of the normal to the graph of f at P. [3]

The normal meets the x-axis at the point Q.

(c) Write down the coordinates of Q. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Differentiate	$f'(x) = 2x - 5$	(A1)
Step 2 Substitute $x = 3$	$f'(3) = 1$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Normal gradient: $-\dfrac{1}{f'(a)}$	gradient of the normal $= -\dfrac{1}{1} = -1$	(A1)
	Write it first, and circle it: this is the step that is forgotten.	
Step 2 The line through P	$y - (-2) = -1(x - 3)$	M1
	M0 if the tangent gradient 1 is used.	
Step 3 Simplify	$y = -x + 1$	A1
	No form is asked here: $x + y - 1 = 0$ is equally correct.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $y = 0$ in the line	$Q(1, 0)$	A1
	"Write down": no working is needed. Follow through from their normal in (b).	

EXAM-STYLE QUESTION 5

Paper 1 · no calculator · [Maximum mark: 7]

Let $f(x) = x^2 + 2x - 3$. The point $P(0, -3)$ lies on the graph of f.

(a) Find the equation of the normal to the graph of f at P. [4]

The normal meets the graph of f again at the point Q.

(b) Find the coordinates of Q. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Gradient of the tangent at P	$f'(x) = 2x + 2,\ f'(0) = 2$	(A1)
Step 2 Normal gradient	gradient of the normal $-\frac{1}{2}$	(A1)
Step 3 The line through P	$y + 3 = -\frac{1}{2}(x - 0)$	M1
Step 4 Simplify	$y = -\frac{1}{2}x - 3$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Curve = line	$x^2 + 2x - 3 = -\frac{1}{2}x - 3$	M1
	Equate the curve and their normal.	
Step 2 One root is P: factorise	$x\left(x + \frac{5}{2}\right) = 0$	(A1)
	$x = 0$ is P: it is always a root, which helps to factorise.	
Step 3 Keep the other root	$Q\left(-\frac{5}{2}, -\frac{7}{4}\right)$	A1
	Accept $(-2.5, -1.75)$. Giving P again earns M1(A1)A0.	

EXAM-STYLE QUESTION 6

Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = x^3 - 12x$.

(a) Find the coordinates of the points on the graph of f at which the tangent is horizontal. [3]

(b) Write down the equations of the normals to the graph of f at these points. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Solve $f'(x) = 0$	$f'(x) = 3x^2 - 12 = 0$	M1
	$x = \pm 2$	A1
Step 2 y from the curve	$(2, -16)$ and $(-2, 16)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Vertical normal	$x = 2$	A1
	The tangent is horizontal, so the normal is vertical: there is no $-\frac{1}{f'(a)}$ here.	
	$x = -2$	A1

C Working back from a gradient

EXAM-STYLE QUESTION 7

Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = x^3 - 6x^2 + 9x + 2$.

(a) Find $f'(x)$. [2]

(b) Find the coordinates of the point on the graph of f at which the tangent is parallel to the line $y = -3x + 7$. [4]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Differentiate	$f'(x) = 3x^2 - 12x + 9$	A2
	A1 for two correct terms out of three.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Parallel: equal gradients	parallel, so equal gradients: $3x^2 - 12x + 9 = -3$	M1
	The gradient of $y = -3x + 7$ is read directly: it is already in the form $y = mx + c$.	
Step 2 Solve	$3x^2 - 12x + 12 = 0$, $(x-2)^2 = 0$	(A1)
	$x = 2$	A1
	A double root: only one point of the curve has this gradient.	
Step 3 y from the curve	$(2, 4)$	A1
	$f(2) = 8 - 24 + 18 + 2 = 4$, from f and not from the line.	

EXAM-STYLE QUESTION 8

Paper 1 · no calculator · [Maximum mark: 5]

The tangent to the curve $y = x^2 + x$ at the point P is perpendicular to the line $x + 3y = 6$.

(a) Find the coordinates of P. [3]

(b) Find the equation of this tangent. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Gradient of the line, then perpendicular	$x + 3y = 6$ is $y = -\frac{1}{3}x + 2$: gradient $-\frac{1}{3}$, so the tangent has gradient 3	(M1)
	Rewrite the line as $y = mx + c$ before reading its gradient.	
Step 2 Solve $f'(x) = 3$	$2x + 1 = 3$	M1
Step 3 y from the curve	$P(1, 2)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The tangent	$y - 2 = 3(x - 1)$	M1
Step 2 Simplify	$y = 3x - 1$	A1

EXAM-STYLE QUESTION 9

Paper 1 · no calculator · [Maximum mark: 4]

The line $y = 2x + k$ is a tangent to the curve $y = x^2 - 4x + 7$. Find the value of k.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Same gradient	equal gradients: $2x - 4 = 2$	M1
	"Is a tangent to" hides two equations: the gradient one gives x at once.	
Step 2 Solve	$x = 3$	A1
Step 3 Same point	equal values at $x = 3$: $9 - 12 + 7 = 6 + k$	M1
Step 4 Solve for k	$k = -2$	A1

Alternative: $x^2 - 6x + 7 - k = 0$ has a double root, so $36 - 4(7 - k) = 0$. Same marks.

EXAM-STYLE QUESTION 10

Paper 1 · no calculator · [Maximum mark: 5]

The curve $y = ax^2 + bx$, where a and b are constants, passes through the point $(1, 4)$. The tangent to the curve at this point is $y = 6x - 2$. Find the value of a and the value of b.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Same point	same point: $a + b = 4$	M1
	The point is on the curve: substitute it into the curve, not into the tangent.	
Step 2 Same gradient	$\frac{dy}{dx} = 2ax + b$, same gradient: $2a + b = 6$	M1A1
Step 3 Solve the two equations	$a = 2,\ b = 2$	A1A1
	Two unknowns need two equations: the two conditions of the property above.	

D With the GDC (Paper 2)

EXAM-STYLE QUESTION 11

Paper 2 · calculator · [Maximum mark: 6]

Let $f(x) = \ln(x^2 + 1)$.

(a) Find the gradient of the graph of f at the point where $x = 1.5$. [2]

(b) Find the equation of the tangent to the graph of f at this point. Give your answer in the form $y = mx + c$. [2]

(c) Find the equation of the normal to the graph of f at this point. Give your answer in the form $y = mx + c$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Gradient at $x = 1.5$ on the GDC	use of GDC, or $f'(x) = \dfrac{2x}{x^2+1}$	(M1)
	On Paper 2 the derivative is not asked for: the GDC value is enough.	
	$f'(1.5) = 0.923 \quad (0.923076\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Point stored, point-gradient form	$y - 1.17866 = 0.923077(x - 1.5)$	(M1)
	$f(1.5) = \ln 3.25 = 1.17866\ldots$, stored, not rounded.	
Step 2 Write $y = mx + c$	$y = 0.923x - 0.206$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Normal gradient	gradient $-\dfrac{1}{0.923077} = -1.08333\ldots$	(M1)
Step 2 Write $y = mx + c$	$y = -1.08x + 2.80$	A1
	Accept $-1.08x + 2.8$. Follow through from their (a).	

EXAM-STYLE QUESTION 12

Paper 2 · calculator · [Maximum mark: 4]

Let $f(x) = x^2 e^{-x}$, for $x \geq 0$. Find the x-coordinates of the points on the graph of f at which the gradient of the tangent is -0.1.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write $f'(x) = -0.1$	$f'(x) = -0.1$ (seen or implied by a sketch of f' and $y = -0.1$)	(M1)
	Write the equation even on Paper 2: it earns the method mark.	
Step 2 Intersections on the GDC	$x = 2.48 \quad (2.48210\ldots)$	A1
	$x = 5.02 \quad (5.02270\ldots)$	A1
	Two answers: the graph of f' crosses $y = -0.1$ twice for $x \geq 0$.	
Step 3 Keep the domain	both values, $x = 2.48$ and $x = 5.02$, and no others	A1
	$x = -0.0466$ is outside the domain $x \geq 0$: A0 if it is given as well.	

EXAM-STYLE QUESTION 13 Paper 2 · calculator · [Maximum mark: 6]

Let $f(x) = e^x - 2x$. The point $P(0, 1)$ lies on the graph of f.

(a) Find the equation of the normal to the graph of f at P. [3]

The normal meets the graph of f again at the point Q.

(b) Find the coordinates of Q. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Gradient of the tangent at P	$f'(x) = e^x - 2,\ f'(0) = -1$	(A1)
Step 2 Normal gradient	normal gradient 1	(A1)
Step 3 The line	$y = x + 1$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Curve = line	$e^x - 2x = x + 1$ (or a sketch of both graphs)	(M1)
Step 2 One root is P	$x = 0$ is P, so the other solution	(R1)
	On the GDC, start the search after $x = 0$.	
Step 3 The other root on the GDC	$Q(1.90, 2.90)$	A1
	$x = 1.90381\ldots$, $y = x + 1$.	

V WHICH METHOD? READ THE QUESTION

5.4 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"the tangent … at the point where $x = a$"	a point the curve f	$f(a)$ and $f'(a)$, then $y - f(a) = f'(a)(x - a)$ P1
"the normal … at the point P"	a point the curve f	$f'(a)$, then $-\frac{1}{f'(a)}$ **first**, then the line through P P2
"the tangent is horizontal"	a gradient: 0	solve $f'(x) = 0$; the normals are vertical: $x = a$ N1
"the tangent is parallel to $y = mx + c$" *"the gradient is m"*	a gradient m	solve $f'(x) = m$; then $y = f(x)$ P3
"the tangent is perpendicular to …"	a gradient m perpendicular: $-\frac{1}{m}$	rewrite the line as $y = mx + c$; solve $f'(x) = -\frac{1}{m}$; then $y = f(x)$ P3
"the line … is a tangent to the curve" *"touches"*	a line not the point	two equations at $x = t$: $f(t) = mt + c$ **and** $f'(t) = m$ P4
"the tangent at (p, q) is $y = mx + c$", unknown constants	a point a gradient two unknowns	two equations: the point is on the curve, $f'(p) = m$ P4
"the normal meets the curve again at Q"	one root already: $x = a$	solve $f(x) =$ the line; set aside $x = a$, keep the other root
"meets the axes at A and B"	a line	$y = 0$ and $x = 0$ in the line, not in f
Paper 2, a decimal value of x, or f too hard to differentiate	the GDC is allowed	Part V: the gradient, the tangent, the intersection on the GDC

In the exam 1 *command terms*

Find: working is marked. **Write down**: no working needed. **Show that**: every step, the last line is the answer given. **Hence**: use the previous part. *Give your answer in the form $y = mx + c$*: an unsimplified line does not earn the last mark. No form asked: any correct equation of the line. Paper 2: three significant figures.

VI ON THE GDC: PAPER 2

SL 5.4 · NumWorks · every calculator move of this topic

1 The gradient at a point

Calculation → Toolbox → Calculus → Derivative

Type the function, then the value of x. Then sto→ A: the full value is kept for the next steps. **On paper:** $f'(1.5) = 0.923$ (GDC). That line earns the (M1)A1 of "find the gradient".

CALCULATION

```
diff(ln(x^2+1),x,1.5)
0.9230769231
```

2 The tangent at a point

Grapher → OK on the curve → Find → Tangent

Move the cursor to $x = 1.5$ (type the value): the GDC gives the equation. **On paper:** the point, the gradient, then $y = 0.923x - 0.206$. Without this menu: $c = f(a) - a\,f'(a)$ with the stored values.

GRAPH · TANGENT

```
f(x)=ln(x^2+1)
x=1.5 y=1.178655
y=0.9230769x-0.2059604
```

3 The normal at a point

Calculation → -1/A → sto→ → B

There is no "normal" menu: the gradient of the normal is $-1/f'(a)$, computed from the stored $f'(a)$. **On paper:** gradient -1.08, then $y = -1.08x + 2.80$.

CALCULATION

```
-1/A
-1.083333333
1.178655-1.5*B
2.803655
```

4 A given gradient: where is $f'(x) = m$?

Grapher → g(x) = derivative of f → Find → X given Y

Plot $g(x) = f'(x)$ as a second function (toolbox → Derivative), then Find → X given Y with the value m; the right arrow for the next one. **On paper:** "$f'(x) = m$" written as an equation, then each value of x.

GRAPH · INTERSECTION

```
f(x)=x^2*e^(-x)
f'(x) plotted, g(x)=-0.1
x=2.482109 y=-0.1
```

5 A line meets the curve again

Grapher → OK on the curve → Find → Intersection

Enter f and the line. Choose the x-window so that the known point is outside it, or move the cursor past it. **On paper:** the equation $f(x) =$ the line, "$x = a$ is P", then the other value.

GRAPH · INTERSECTION

```
f(x)=e^x-2x
g(x)=x+1
X: min 0.5, max 3
x=1.903814 y=2.903814
```

ANSWERS TO THE APPLY QUESTIONS

I.1 $y = 10x - 16$

I.2 $y = -\frac{2}{3}x + 4$

I.3 $y = x - 1$

I.4 $y = -x + \pi$

I.5 $(2, 0), (0, 6);\ 6$

II.1 $y = -\frac{1}{3}x - \frac{4}{3}$

II.2 $y = -2x + 12$

II.3 $y = -4;\ x = 3$

II.4 $y = \frac{1}{4}x;\ Q\left(\frac{17}{4}, \frac{17}{16}\right)$

III.1 $(2, 9), (-2, -7)$

III.2 $(4, 8)$

III.3 $(1, 2)$

III.4 $k = 3$

III.5 $a = 2, b = 1$

III.6 $y = 0.824x + 0.824$

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