Marked example 3

Paper 1 style · No calculator

3 6 marks SL 5.4 ●●● Demanding

Let $f(x) = x^2 - 2x + 3$ and $g(x) = k - x^2$, where k is a constant. The graphs of f and g touch at a single point T: at T, the two graphs have a common tangent.

(a) Find the x-coordinate of T and the value of k. **[4]**

(b) Find the equation of the common tangent. **[2]**

MARKSCHEME (a)

recognising that the gradients are equal at T: $2x - 2 = -2x$ **M1**

$x = \frac{1}{2}$ **A1**

recognising that the values are equal at T: $\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 3 = k - \left(\frac{1}{2}\right)^2$ **M1**

$k = \frac{5}{2}$ **A1**

Total 4 marks

ALTERNATIVE METHOD

$x^2 - 2x + 3 = k - x^2$ has exactly one solution, so the discriminant of $2x^2 - 2x + 3 - k = 0$ is zero **M1**

$4 - 8(3 - k) = 0$ **A1**

$k = \frac{5}{2}$ **A1**

$x = \frac{1}{2}$ **A1**

MARKSCHEME (b)

gradient $f'\left(\frac{1}{2}\right) = -1$, point $\left(\frac{1}{2}, \frac{9}{4}\right)$ **(A1)**

$y = -x + \frac{11}{4}$ **A1**

Total 2 marks

Note FT from their values in (a).

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