Exercise 3

Paper 1 style · No calculator

3 6 marks SL 5.4 ●●● Demanding

Let $f(x) = x^2 + 4x + 1$ and $g(x) = k - 2x^2$, where k is a constant. The graphs of f and g touch at a single point T: at T, the two graphs have a common tangent.

(a) Find the x-coordinate of T and the value of k. **[4]**

(b) Find the equation of the common tangent. **[2]**

The markscheme is on the next page: try the question first. Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization.
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MARKSCHEME (a)

recognising that the gradients are equal at T: $2x + 4 = -4x$ **M1**

$x = -\frac{2}{3}$ **A1**

recognising that the values are equal at T: $\frac{4}{9} - \frac{8}{3} + 1 = k - \frac{8}{9}$ **M1**

$k = -\frac{1}{3}$ **A1**

Total 4 marks

ALTERNATIVE METHOD

$x^2 + 4x + 1 = k - 2x^2$ has exactly one solution, so the discriminant of $3x^2 + 4x + 1 - k = 0$ is zero **M1**

$16 - 12(1 - k) = 0$ **A1**

$k = -\frac{1}{3}$ **A1**

$x = -\frac{4}{6} = -\frac{2}{3}$ **A1**

MARKSCHEME (b)

gradient $f'\left(-\frac{2}{3}\right) = \frac{8}{3}$, point $\left(-\frac{2}{3}, -\frac{11}{9}\right)$ **(A1)**

$y = \frac{8}{3}x + \frac{5}{9}$ **A1**

Total 2 marks

Note FT from their values in (a). Accept $8x - 3y + \frac{5}{3} = 0$ or any equivalent form.