Tangents and normals

Skills sheet · SL AA 5.4

KEY FACTS

Tangent at $x = a$: $y - f(a) = f'(a)(x - a)$ P1

Normal at $x = a$: gradient $-\dfrac{1}{f'(a)}$ P2

Parallel: equal gradients. **Perpendicular**: $m_1 m_2 = -1$ P2

$f'(a) = 0$: tangent $y = f(a)$, normal $x = a$ N1

A point gives the gradient, a gradient gives the point F4.

"Touches" hides two equations: equal values and equal gradients.

Paper 2: $f'(a)$ on the GDC, stored, never rounded early X3.

WHAT UNLOCKS

- "the gradient of the curve at..." $\rightarrow$ it is $f'(a)$: differentiate, then substitute.
- "normal" $\rightarrow$ write $-\dfrac{1}{f'(a)}$ *before* anything else, and circle it.
- "meets the curve again" $\rightarrow$ one root of the equation is the point you know: factorise it out.
- "parallel to $x + 4y = 1$" $\rightarrow$ first rewrite the line as $y = -\frac{1}{4}x + \frac{1}{4}$ to read its gradient.

WORKED EXAMPLE ①

Let $f(x) = x^2 + 1$. Find the equation of the normal to the graph of f at the point where $x = 1$. no GDC

$f(1) = 2$, point $(1, 2)$ (A1)

$f'(x) = 2x$, $f'(1) = 2$ (A1)

normal gradient $-\frac{1}{2}$: $y - 2 = -\frac{1}{2}(x - 1)$ M1

$y = -\frac{1}{2}x + \frac{5}{2}$ A1

Method

How do I find a normal?

① Differentiate, substitute $x = a$.

② Turn it over and change its sign: $-\dfrac{1}{f'(a)}$.

③ The point on the curve: $\big(a, f(a)\big)$.

④ $y - f(a) = m(x - a)$, then the form asked.

Using $f'(a)$ for the normal: M0, and every mark after it.

WORKED EXAMPLE ②

The tangent to $y = 3x^2 - 2x$ at the point P is perpendicular to the line $x + 4y = 1$. Find the coordinates of P and the equation of this tangent. no GDC

line: $y = -\frac{1}{4}x + \frac{1}{4}$, gradient $-\frac{1}{4}$ (A1)

tangent gradient 4: $6x - 2 = 4$ M1

$x = 1$, $P(1, 1)$ A1

$y = 4x - 3$ A1

Method

How do I use a given gradient?

① Read the gradient of the line: write it as $y = mx + c$.

② Parallel: m. Perpendicular: $-\dfrac{1}{m}$.

③ Solve $f'(x) =$ that gradient.

④ The y-coordinate comes from f, not from the line.

FLUENCY

●○○ **1.** Tangent to $y = 2x^3 - 5x$ at $x = 1$. no GDC

●○○ **2.** Normal to $y = \dfrac{4}{x}$ at $x = 2$. no GDC

●○○ **3.** Tangent to $y = \sqrt{x}$ at $(9, 3)$, in the form $y = mx + c$. no GDC

●○○ **4.** Point of $y = x^2 - 3x$ where the tangent is parallel to $y = 5x + 1$. no GDC

●○○ **5.** Tangent to $y = e^{2x}$ at $x = 0$, and where it meets the x-axis. no GDC

●○○ **6.** Normal to $y = x^3$ at $(1, 1)$. no GDC

REASONING

●●○ **Question 1.** Let $f(x) = x^3 - 3x^2 + 2$. [no GDC]

(a) Find the coordinates of the points of the graph at which the tangent is horizontal. [4]

(b) Write down the equations of the normals at these points. [2]

●●○ **Question 2.** Let $f(x) = \cos x + \dfrac{x}{3}$, with x in radians. [with GDC]

(a) Find the gradient of the graph of f at $x = 2$. [2]

(b) Find the equation of the tangent at this point, in the form $y = mx + c$. [2]

●●○ **Question 3.** The tangent to the curve $y = x^2 - 2x$ at the point where $x = a$ passes through $(0, -4)$. [no GDC]

(a) Show that the tangent has equation $y = (2a - 2)x - a^2$. [3]

(b) Find the possible values of a, and the points of contact. [3]

PROBLEMS

●●● **Question 4.** The tangent to the curve $y = x^3$ at the point $(1, 1)$ meets the curve again at the point Q. [no GDC]

(a) Find the equation of the tangent. [3]

(b) Find the coordinates of Q. [3]

(c) Find the equation of the normal to the curve at Q. [3]

●●● **Question 5.** Let $f(x) = 2x^2 - 3x + 1$. The point $P(1, 0)$ lies on the graph of f. [no GDC]

(a) Find the equation of the normal at P. [3]

(b) The normal meets the graph again at Q. Find the coordinates of Q and the exact distance PQ. [4]

(c) The tangents at P and at Q meet at R. Find the coordinates of R. [4]

ANSWERS

1 $y' = 6x^2 - 5$, gradient 1 at $(1, -3)$: $y = x - 4$

2 $y' = -\dfrac{4}{x^2} = -1$ at $(2, 2)$, normal gradient 1: $y = x$

3 $y' = \dfrac{1}{2\sqrt{x}} = \dfrac{1}{6}$: $y = \dfrac{1}{6}x + \dfrac{3}{2}$

4 $2x - 3 = 5$: $(4, 4)$

5 $y = 2x + 1$, meets the x-axis at $\left(-\frac{1}{2}, 0\right)$

6 $y' = 3$ at $(1, 1)$, normal gradient $-\frac{1}{3}$: $y = -\frac{1}{3}x + \frac{4}{3}$

Q1 (a) $3x^2 - 6x = 0$: $(0, 2)$ and $(2, -2)$ (b) horizontal tangents, so vertical normals: $x = 0$ and $x = 2$

Q2 (a) $f'(2) = -\sin 2 + \frac{1}{3} = -0.576$ $(-0.575964\ldots)$ (b) $f(2) = 0.250520\ldots$: $y = -0.576x + 1.40$

Q3 (a) $y - (a^2 - 2a) = (2a - 2)(x - a)$ expands to $y = (2a - 2)x - a^2$ (b) $-a^2 = -4$, $a = \pm 2$: $(2, 0)$ and $(-2, 8)$

Q4 (a) $y = 3x - 2$ (b) $x^3 - 3x + 2 = (x - 1)^2(x + 2) = 0$; $x = 1$ is the point of contact, so $Q(-2, -8)$ (c) gradient $-\frac{1}{12}$: $y = -\frac{1}{12}x - \frac{49}{6}$

Q5 (a) $f'(1) = 1$, normal $y = -x + 1$ (b) $2x^2 - 2x = 0$: $Q(0, 1)$, $PQ = \sqrt{2}$ (c) tangents $y = x - 1$ and $y = -3x + 1$: $R\left(\frac{1}{2}, -\frac{1}{2}\right)$

Written and verified by a practising IB mathematics teacher, produced with AI assistance; every result recomputed by software, every figure computed from its function. Not affiliated with or endorsed by the International Baccalaureate Organization. © 2026 Hence