The graphs of f, f' and f''

Skills sheet · SL AA 5.2 · 5.7

KEY FACTS

$f' > 0$: f increasing; $f' < 0$: decreasing D1

f': + to −, maximum; − to +, minimum; touches 0: neither P3

$f'' > 0$: concave up; $f'' < 0$: concave down D3

Inflexion: f'' **changes sign**; f' has a turning point P4

An interval is a set of x, with strict inequalities W1.

A maximum of f' is an inflexion of f W2.

$f''(a) = 0$ is not enough W3.

WHAT UNLOCKS

- "increasing" or "decreasing" → factorise $f'(x)$ and make a sign table.
- a factor $e^{\cdots}$ in f' → it is positive: the sign comes from the other factor.
- the graph of f' is given → above the axis, f rises; crossing the axis, f turns; its own turning points are inflexions of f.
- "justify" → the sign change, in words, before the answer.

WORKED EXAMPLE ①

Find the values of x for which $f(x) = x^3 - 3x + 1$ is increasing. no GDC

$f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$ A1

$f'(x) > 0$ M1

$x < -1$ or $x > 1$ A1

Method

How do I find where f increases?

① Differentiate and factorise.
② Zeros of f' on a number line.
③ The sign of f' between them.
④ Answer in x: $f' > 0$ intervals.

The interval between the zeros is not always the answer: check the sign.

WORKED EXAMPLE ②

Show that the graph of $f(x) = x^3 - 3x^2 + 2$ has a point of inflexion, and find it. no GDC

$f''(x) = 6x - 6 = 0$: $x = 1$ M1A1

$f'' < 0$ for $x < 1$, $f'' > 0$ for $x > 1$: it changes sign R1

$f(1) = 0$: $(1, 0)$ A1

Method

How do I prove an inflexion?

① Differentiate twice.
② Solve $f''(x) = 0$: the candidates.
③ The sign of f'' on each side.
④ A change of sign: inflexion; $y = f(x)$.

FLUENCY

●○○ **1.** $f'(x) = (x-2)(x+5)$: where is f increasing? no GDC

●○○ **2.** Where is $f(x) = x^2 - 6x + 1$ decreasing? no GDC

●○○ **3.** Where is $f(x) = x^3 - 12x$ increasing? no GDC

●○○ **4.** Where is $f(x) = e^{2x} - 4x$ increasing? no GDC

●○○ **5.** Where is $f(x) = x^3 + 3x^2$ concave up? no GDC

●○○ **6.** $f(x) = x^4$: is $(0, 0)$ a point of inflexion? no GDC

REASONING

●●● **Question 1.** Let $f(x) = (x-1)\,e^x$. [no GDC]

(a) Show that $f'(x) = x\,e^x$, and find where f is decreasing. [4]

(b) Find the exact coordinates of the point of inflexion. [3]

●●● **Question 2.** A cubic function f has a local maximum at $(-2, 5)$, a local minimum at $(3, -1)$, and no other stationary point. [no GDC]

(a) Write down the solutions of $f'(x) = 0$, and the values of x for which $f'(x) < 0$. [3]

(b) Write down the x-coordinate of the point of inflexion. [1]

●●● **Question 3.** The derivative of a function f is $f'(x) = x^2(x-3)$. [no GDC]

(a) Find the x-coordinates of the stationary points of f and their nature. [4]

(b) Find the x-coordinates of the points of inflexion of f. [3]

PROBLEMS

●●● **Question 4.** Let $f(x) = x^4 - 6x^2 + 8x$. [no GDC]

(a) Show that $f'(x) = 4(x-1)^2(x+2)$. [3]

(b) Find the nature of each stationary point. Justify your answer. [4]

(c) Find the coordinates of the points of inflexion. [4]

●●● **Question 5.** Let $f(x) = x^3 + kx^2 + 3x$, where k is a constant. The graph of f has a point of inflexion at $x = 2$. [no GDC]

(a) Find the value of k. [2]

(b) Find the exact values of x for which f is decreasing. [4]

ANSWERS

1 $x < -5$ or $x > 2$

2 $f'(x) = 2x - 6 < 0$: $x < 3$

3 $3(x-2)(x+2) > 0$: $x < -2$ or $x > 2$

4 $2\,\mathrm{e}^{2x} - 4 > 0$: $x > \dfrac{\ln 2}{2}$

5 $6x + 6 > 0$: $x > -1$

6 No: $f''(x) = 12x^2 \geq 0$ on both sides, no change of sign

Q1 (a) $\mathrm{e}^x + (x-1)\mathrm{e}^x = x\,\mathrm{e}^x$; decreasing for $x < 0$ (b) $f''(x) = (x+1)\mathrm{e}^x$ changes sign at $x = -1$: $(-1,\ -2\mathrm{e}^{-1})$

Q2 (a) $x = -2$, $x = 3$; $-2 < x < 3$ (b) $x = \dfrac{1}{2}$, halfway between the turning points of a cubic

Q3 (a) $x = 0$: $f' \leq 0$ on both sides, neither (a stationary inflexion); $x = 3$: $-$ to $+$, minimum (b) $f''(x) = 3x^2 - 6x$: $x = 0$ and $x = 2$

Q4 (a) $4x^3 - 12x + 8 = 4(x-1)(x^2 + x - 2) = 4(x-1)^2(x+2)$ (b) $x = -2$: $-$ to $+$, minimum; $x = 1$: $+$ on both sides, neither (c) $f''(x) = 12x^2 - 12$: $(-1,\ -13)$ and $(1,\ 3)$

Q5 (a) $f''(x) = 6x + 2k$, $12 + 2k = 0$: $k = -6$ (b) $3x^2 - 12x + 3 < 0$: $2 - \sqrt{3} < x < 2 + \sqrt{3}$

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