Composite and inverse functions

Two machines in a row make a composite function: the output of the first is the input of the second. An inverse function is the machine that runs backwards, and its graph is the mirror image of the first in the line $y = x$.

I COMPOSITE FUNCTIONS

SL 2.5 · $(f \circ g)(x) = f(g(x))$, its domain

I.1 A composite at a number

Definition 1 *composite function*

$(f \circ g)(x) = f(g(x))$: apply g first, then f to the result. It is defined for the x of the domain of g for which $g(x)$ is in the domain of f.

Example 1

Let $f(x) = x^2$ and $g(x) = x + 3$. Find $(f \circ g)(1)$ and $(g \circ f)(1)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Inside first: $g(1)$	$g(1) = 4$
Step 2 Then f of it	$(f \circ g)(1) = f(4) = 16$
Step 3 Other order: $f(1)$, then g	$(g \circ f)(1) = g(1) = 4$
Two different answers: the order matters.	

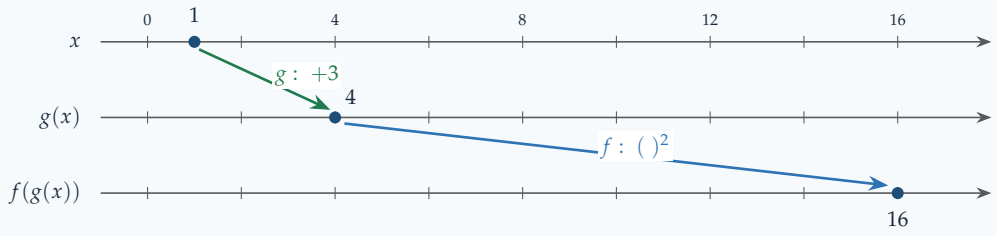


Figure 1 — Example 1 on the diagram: $1 \to g(1) = 4 \to f(4) = 16$; in the other order, $1 \to f(1) = 1 \to g(1) = 4$.

APPLY

1 $f(x) = 2x + 1, g(x) = x^2$: find $(f \circ g)(3)$ and $(g \circ f)(3)$.

2 $f(x) = 10 - x, g(x) = 3x$: find $(f \circ g)(2)$.

3 $f(x) = \sqrt{x}, g(x) = x + 12$: find $(f \circ g)(4)$.

I.2 A composite as an expression

Watch out 1

$(f \circ g)(x)$ is not the product $f(x) \times g(x)$, and $f \circ g$ is usually not $g \circ f$. Read $f(g(x))$ from the inside out.

Example 2

Let $f(x) = 3x - 1$ and $g(x) = x^2 + 2$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $f \circ g$: put $g(x)$ in place of x in f	$3(x^2 + 2) - 1$
Step 2 Simplify	$(f \circ g)(x) = 3x^2 + 5$
Step 3 $g \circ f$: put $f(x)$ in place of x in g	$(3x - 1)^2 + 2$
Step 4 Expand and simplify	$(g \circ f)(x) = 9x^2 - 6x + 3$

APPLY

4 $f(x) = x + 4$, $g(x) = 5x$: find $(f \circ g)(x)$ and $(g \circ f)(x)$.

5 $f(x) = x^2$, $g(x) = 1 - 2x$: find $(f \circ g)(x)$.

6 $f(x) = \dfrac{2}{x}$, $g(x) = x - 3$: find $(f \circ g)(x)$.

I.3 Solving $(f \circ g)(x) = k$

Example 3

Let $f(x) = 2x - 3$ and $g(x) = x^2$. Solve $(f \circ g)(x) = 29$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the composite	$(f \circ g)(x) = 2x^2 - 3$
Step 2 Set it equal to k	$2x^2 - 3 = 29$
Step 3 Solve: both signs	$x^2 = 16$: $x = 4$ or $x = -4$

APPLY

7 $f(x) = x - 5$, $g(x) = x^2$: solve $(f \circ g)(x) = 31$.

8 $f(x) = 3x$, $g(x) = x + 2$: solve $(f \circ g)(x) = 24$.

I.4 The domain of a composite

Example 4

Let $f(x) = \sqrt{x}$, for $x \geq 0$, and $g(x) = x - 7$. Find $(f \circ g)(x)$ and the domain of $f \circ g$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the composite	$(f \circ g)(x) = \sqrt{x-7}$
Step 2 $g(x)$ must be in the domain of f	$x - 7 \geq 0$
Step 3 Solve	domain $x \geq 7$

APPLY

9 $f(x) = \ln x$, $g(x) = x + 2$: domain of $f \circ g$.

10 $f(x) = \dfrac{1}{x}$, $g(x) = x - 1$: domain of $f \circ g$.

11 $f(x) = \sqrt{x}$, $g(x) = 6 - 2x$: domain of $f \circ g$.

I.5 The identity

Property 1 *the identity*

The **identity** is $x \mapsto x$. A function and its inverse undo each other: $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

Example 5

Let $f(x) = 4x - 3$ and $g(x) = \dfrac{x+3}{4}$. Show that $(f \circ g)(x) = x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Put $g(x)$ into f	$(f \circ g)(x) = 4 \times \dfrac{x+3}{4} - 3$
Step 2 Simplify to x	$= x + 3 - 3 = x$

APPLY

12 $f(x) = 2x + 6$, $g(x) = \frac{x}{2} - 3$: show that $(f \circ g)(x) = x$.

13 $f(x) = x^3$, $g(x) = \sqrt[3]{x}$: find $(g \circ f)(x)$.

II INVERSE FUNCTIONS

SL 2.2 · 2.5 · f^{-1}, its graph, its domain and range

II.1 The inverse undoes f

Definition 2 *inverse function*

If $f(a) = b$, then $f^{-1}(b) = a$. f^{-1} exists when f is **one-to-one** (each value comes from one x only). Domain of f^{-1} = range of f; range of f^{-1} = domain of f.

Example 6

Let $f(x) = 2x + 7$. Find $f^{-1}(15)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $f^{-1}(15)$ is the x with $f(x) = 15$	$2x + 7 = 15$
Step 2 Solve	$x = 4$
Step 3 Check with f	$f(4) = 15$, so $f^{-1}(15) = 4$

APPLY

1 $f(x) = 5x - 2$: find $f^{-1}(13)$.

2 $f(x) = x^3$: find $f^{-1}(-8)$.

3 $f(x) = x^2 + 3$, $x \geq 0$: find $f^{-1}(19)$.

II.2 Finding $f^{-1}(x)$

Property 2 *finding the inverse*

Write $y = f(x)$, exchange x and y, make y the subject: that y is $f^{-1}(x)$.

Example 7

Let $f(x) = 4x + 1$. Find $f^{-1}(x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write $y = f(x)$, exchange x and y	$x = 4y + 1$
Step 2 Make y the subject	$y = \dfrac{x - 1}{4}$
Step 3 Name it	$f^{-1}(x) = \dfrac{x - 1}{4}$

APPLY

4 $f(x) = 7 - 2x$: find $f^{-1}(x)$.

5 $f(x) = \frac{x}{3} + 2$: find $f^{-1}(x)$.

6 $f(x) = x^3 - 1$: find $f^{-1}(x)$.

II.3 The inverse of a rational function

Example 8

Let $f(x) = \dfrac{3x-1}{x+2}$, for $x \neq -2$. Find $f^{-1}(x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Exchange x and y	$x = \dfrac{3y-1}{y+2}$
Step 2 Multiply out the denominator	$xy + 2x = 3y - 1$
Step 3 Collect the y terms on one side Factorise y out.	$xy - 3y = -2x - 1,\ y(x-3) = -2x - 1$
Step 4 Divide	$f^{-1}(x) = \dfrac{2x+1}{3-x}$

APPLY

7 $f(x) = \dfrac{1}{x-2}$: find $f^{-1}(x)$.

8 $f(x) = \dfrac{2x}{x+5}$: find $f^{-1}(x)$.

II.4 The graph of the inverse

Property 3 *the graph of the inverse*

The graph of f^{-1} is the reflection of the graph of f in the line $y = x$: (a, b) becomes (b, a). An increasing f meets f^{-1} on the line $y = x$.

Watch out 2

$f^{-1}(x)$ is not $\dfrac{1}{f(x)}$.

Example 9

Let $f(x) = 2x - 3$. The point $(3, 3)$ is on its graph; so is $(5, 7)$. Write down a point of the graph of f^{-1}, then solve $f(x) = f^{-1}(x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Swap the coordinates	$(7, 5)$ is on the graph of f^{-1}
Step 2 f is increasing: they meet on $y = x$	$2x - 3 = x$
Step 3 Solve	$x = 3$

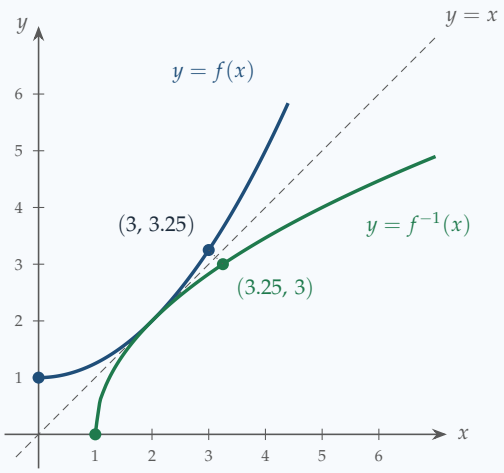


Figure 2 — $f(x) = \frac{x^2}{4} + 1$, for $x \geq 0$, and $f^{-1}(x) = 2\sqrt{x-1}$, for $x \geq 1$: each point (a, b) of one has its mirror (b, a) on the other.

APPLY

9 $(-2, 5)$ is on the graph of f: a point of the graph of f^{-1}.

10 $f(x) = 4x - 9$: solve $f(x) = f^{-1}(x)$.

11 $f(x) = \frac{x}{2} + 3$: solve $f(x) = f^{-1}(x)$.

II.5 Restricting the domain

Example 10

Let $f(x) = (x+1)^2$, for $x \geq -1$. Find $f^{-1}(x)$, with its domain and range.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Range of f: the square is ≥ 0	$f(x) \geq 0$
Step 2 Exchange x and y	$x = (y+1)^2$
Step 3 Positive root, because $y \geq -1$ No $\pm$ in the answer.	$y + 1 = \sqrt{x},\ f^{-1}(x) = \sqrt{x} - 1$
Step 4 Domain and range swap	domain $x \geq 0$, range $f^{-1}(x) \geq -1$

APPLY

12 $f(x) = x^2 - 4$, $x \geq 0$: $f^{-1}(x)$ and its domain.

13 $f(x) = (x-3)^2$, $x \geq 3$: $f^{-1}(x)$ and its range.

II.6 An inverse value on the GDC

Example 11

Let $f(x) = x^3 + x$. Find $f^{-1}(4)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the equation $f(x) = 4$	$x^3 + x = 4$
Step 2 Intersection of $y = f(x)$ and $y = 4$ on the GDC	$x = 1.37879\ldots$
Step 3 Answer to 3 s.f.	$f^{-1}(4) = 1.38$

APPLY

14 $f(x) = e^x + x$: find $f^{-1}(3)$.

15 $f(x) = x + \ln x$: find $f^{-1}(4)$.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Composite functions

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = 2x - 1$ and $g(x) = x^2 + 3$.

(a) Find $(f \circ g)(1)$. [2]

(b) Find $(g \circ f)(x)$. Give your answer in the form $ax^2 + bx + c$. [2]

(c) Solve $(f \circ g)(x) = 21$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Inside first: $g(1)$	$g(1) = 4$ *Inside first.*	(A1)
Step 2 Then f of it	$f(4) = 7$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Put $f(x)$ into g	$(2x - 1)^2 + 3$ *$f(x)$ goes into g.*	(M1)
Step 2 Expand and simplify	$= 4x^2 - 4x + 4$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the composite, set it equal to 21	$(f \circ g)(x) = 2x^2 + 5 = 21$	(M1)
Step 2 Solve: both signs	$x = \pm 2\sqrt{2}$ *Both values.*	A1

EXAM-STYLE QUESTION 2

Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = \ln x$, for $x > 0$, and $g(x) = 3 - x$, for $x \in \mathbb{R}$.

(a) Write down $(f \circ g)(x)$. [1]

(b) Find the domain of $f \circ g$. [2]

(c) Solve $(f \circ g)(x) = 0$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Put $g(x)$ into f	$\ln(3 - x)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $g(x)$ must suit ln: > 0	$3 - x > 0$ *The output of g must be in the domain of f.*	(M1)
Step 2 Solve	$x < 3$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\ln u = 0$ when $u = 1$	$3 - x = 1$ *$\ln u = 0$ when $u = 1$.*	(M1)
Step 2 Solve	$x = 2$	A1

EXAM-STYLE QUESTION 3 Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = 3x - 5$.

(a) Find $f^{-1}(x)$. [2]

(b) Find $f^{-1}(7)$. [1]

(c) Solve $f(x) = f^{-1}(x)$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Exchange x and y	$x = 3y - 5$ *Exchange x and y.*	(M1)
Step 2 Make y the subject	$f^{-1}(x) = \dfrac{x+5}{3}$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute in f^{-1}	4 *Check: $f(4) = 7$.*	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the equation	$3x - 5 = \dfrac{x+5}{3}$	M1
Step 2 Clear the fraction	$9x - 15 = x + 5,\ 8x = 20$	(A1)
Step 3 Solve	$x = \dfrac{5}{2}$ *Where the graphs meet, on the line $y = x$.*	A1

EXAM-STYLE QUESTION 4

Paper 1 · no calculator · [Maximum mark: 7]

Let $f(x) = (x-2)^2 + 1$, for $x \geq 2$.

(a) Write down the range of f. [1]

(b) Find $f^{-1}(x)$. [3]

(c) Write down the domain and the range of f^{-1}. [2]

(d) Explain why the domain of f is restricted to $x \geq 2$. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Range from the vertex	$f(x) \geq 1$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Exchange x and y	$x = (y-2)^2 + 1$	(M1)
Step 2 Positive root	$y - 2 = \sqrt{x-1}$ *The positive root, because $y \geq 2$.*	(A1)
Step 3 Make y the subject	$f^{-1}(x) = 2 + \sqrt{x-1}$ *$\pm$ in the final answer: A0.*	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Domain of f^{-1} = range of f	domain $x \geq 1$ *The range of f.*	A1
Step 2 Range of f^{-1} = domain of f	range $f^{-1}(x) \geq 2$ *The domain of f.*	A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Give the reason in words	on $x \geq 2$, f is one-to-one (on $\mathbb{R}$ two values of x give the same $f(x)$)	R1

EXAM-STYLE QUESTION 5 Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = \dfrac{2x+1}{x-3}$, for $x \neq 3$.

(a) Find $f^{-1}(x)$. [3]

(b) Write down the domain of f^{-1}. [1]

(c) Hence write down the range of f. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Exchange x and y	$x = \dfrac{2y+1}{y-3}$	(M1)
Step 2 Collect the y terms on one side	$xy - 3x = 2y + 1 \Rightarrow y(x-2) = 3x+1$	(A1)
	Collect the terms in y on one side.	
Step 3 Divide	$f^{-1}(x) = \dfrac{3x+1}{x-2}$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The denominator $\neq 0$	$x \neq 2$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Range of f = domain of f^{-1}	$f(x) \neq 2$	A1
	The range of f is the domain of f^{-1}.	

C With the GDC (Paper 2)

EXAM-STYLE QUESTION 6

Paper 2 · calculator · [Maximum mark: 5]

Let $f(x) = e^{0.5x} + x$, for $x \in \mathbb{R}$, and $g(x) = \ln x$, for $x > 0$.

(a) Explain why f has an inverse. [1]

(b) Find $f^{-1}(5)$. [2]

(c) Find $(g \circ f)(2)$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Give the reason in words	f is increasing, so it is one-to-one	R1
	Or: $f'(x) = 0.5e^{0.5x} + 1 > 0$.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the equation $f(x) = 5$	$e^{0.5x} + x = 5$	(M1)
Step 2 Solve on the GDC	$2.12 \quad (2.11739\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Inside first: $f(2)$	$f(2) = e + 2 = 4.71828\ldots$	(A1)
Step 2 Then g of it	$\ln(4.71828\ldots) = 1.55 \quad (1.55144\ldots)$	A1

EXAM-STYLE QUESTION 7

Paper 2 · calculator · [Maximum mark: 5]

Let $f(x) = 2^x$ and $g(x) = x^2 - 3$.

(a) Find $(f \circ g)(1.5)$. [2]

(b) Solve $(f \circ g)(x) = 5$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Inside first: $g(1.5)$	$g(1.5) = -0.75$	(A1)
Step 2 Then f of it	$2^{-0.75} = 0.595 \quad (0.594603\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the equation	$2^{x^2-3} = 5$	(M1)
	On the GDC: $y = 2^{x^2-3}$ *and* $y = 5$.	
Step 2 Solve on the GDC	$x = -2.31 \quad (-2.30693\ldots)$	A1
	$x = 2.31 \quad (2.30693\ldots)$	A1

IV WHICH METHOD? READ THE QUESTION

2.5 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"find $(f \circ g)(a)$"	inside first	$g(a)$, then f of it D1
"find $(f \circ g)(x)$"	$g(x)$ goes into f	replace every x of f by $g(x)$; simplify D1
"the domain of $f \circ g$"	$g(x)$ must suit f	solve the condition of f on $g(x)$ D1
"find $f^{-1}(x)$"	exchange x and y	$x = f(y)$, then $y = \ldots$ P2
"find $f^{-1}(a)$"	$f(x) = a$	solve $f(x) = a$, or use $f^{-1}(x)$ D2
"domain and range of f^{-1}"	they swap	domain of f^{-1} = range of f, and back D2
"sketch f^{-1}"	mirror in $y = x$	each (a, b) becomes (b, a); asymptotes swap too P3

In the exam 1 *command terms*

Find $f^{-1}(x)$: an expression in x. **Write down the domain**: the values of x, with the inequality sign. **Hence**: use the inverse just found. **Explain why**: a reason in words, such as "f is one-to-one".

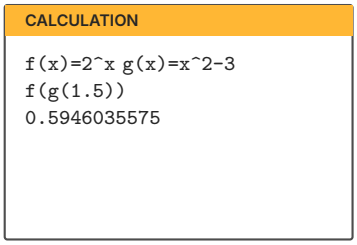
V ON THE GDC: PAPER 2

SL 2.2 · 2.5 · NumWorks · every calculator move of this topic

1 A value of a composite

Calculation → f(g(1.5)) → EXE

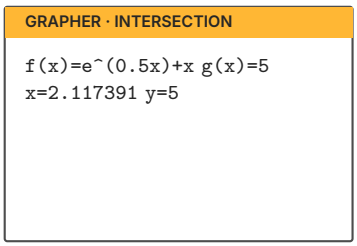
Define f and g in the Functions app, then type $f(g(1.5))$ in Calculation. **On paper:** $g(1.5) = -0.75$, $f(-0.75) = 0.595$.



2 An inverse value: solve $f(x) = k$

Grapher → f(x), y = 5 → Find → Intersection

$f^{-1}(5)$ is the x with $f(x) = 5$: draw $y = f(x)$ and $y = 5$, then the intersection. **On paper:** "$e^{0.5x} + x = 5$", then $f^{-1}(5) = 2.12$.



I.1 $19;\ 49$

I.2 4

I.3 4

I.4 $5x+4;\ 5x+20$

I.5 $4x^2-4x+1$

I.6 $\dfrac{2}{x-3}$

I.7 $x=\pm 6$

I.8 $x=6$

I.9 $x>-2$

I.10 $x\neq 1$

I.11 $x\leq 3$

I.12 $2\left(\frac{x}{2}-3\right)+6=x$

I.13 x

II.1 3

II.2 -2

II.3 4

II.4 $\dfrac{7-x}{2}$

II.5 $3x-6$

II.6 $\sqrt[3]{x+1}$

II.7 $\dfrac{2x+1}{x}$

II.8 $\dfrac{5x}{2-x}$

II.9 $(5,\ -2)$

II.10 $x=3$

II.11 $x=6$

II.12 $\sqrt{x+4};\ x\geq -4$

II.13 $3+\sqrt{x};\ f^{-1}(x)\geq 3$

II.14 0.792

II.15 2.93

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