Composite and inverse functions

Skills sheet · SL AA 2.2 · 2.5

KEY FACTS

$(f \circ g)(x) = f(g(x))$: g first D1
f^{-1} undoes f; domain and range swap D2
Inverse: exchange x and y, make y the subject P2
The graph of f^{-1}: reflect in $y = x$ P3

$f \circ g \neq g \circ f$ in general W1.
$f^{-1}(x) \neq \dfrac{1}{f(x)}$ W2.
One-to-one first, or restrict the domain W2.

WHAT UNLOCKS

- "$(f \circ g)(a)$" $\rightarrow$ a number: $g(a)$ first.
- "$f^{-1}(a)$" $\rightarrow$ solve $f(x) = a$.
- "domain of f^{-1}" $\rightarrow$ the range of f.
- "$f(x) = f^{-1}(x)$" for an increasing f $\rightarrow$ the graphs meet on $y = x$.

WORKED EXAMPLE ①

Let $f(x) = x^2$ and $g(x) = x - 4$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. no GDC

$(f \circ g)(x) = f(x-4) = (x-4)^2$ A1
$(g \circ f)(x) = g\left(x^2\right) = x^2 - 4$ A1

Method
How do I compose?
① Find the inside function.
② Put it in place of every x of the outside one.
③ Simplify.
Inside first, outside last.

WORKED EXAMPLE ②

Find the inverse of $f(x) = \dfrac{x+1}{4}$. no GDC

$x = \dfrac{y+1}{4}$ (M1)
$f^{-1}(x) = 4x - 1$ A1

Method
How do I find f^{-1}?
① $y = f(x)$.
② Exchange x and y.
③ Make y the subject.
④ Check: $f\left(f^{-1}(x)\right) = x$.

FLUENCY

●○○ **1.** $f(x) = 2x$, $g(x) = x + 5$: $(f \circ g)(3)$ no GDC
●○○ **2.** The same f and g: $(g \circ f)(3)$ no GDC
●○○ **3.** The inverse of $x - 7$ no GDC
●○○ **4.** The inverse of $4x$ no GDC
●○○ **5.** The inverse of $x^3 + 1$ no GDC
●○○ **6.** $f(2) = 9$: write down $f^{-1}(9)$ no GDC

REASONING

●●● **Question 1.** Let $f(x) = x^2 + 1$ and $g(x) = 2x - 3$. [no GDC]

(a) Find $(f \circ g)(x)$. [2]

(b) Solve $(f \circ g)(x) = 10$. [3]

●●● **Question 2.** Let $f(x) = \dfrac{3x-2}{x+1}$, for $x \neq -1$. [no GDC]

(a) Find $f^{-1}(x)$. [3]

(b) Write down the domain of f^{-1}. [1]

●●● **Question 3.** Let $f(x) = e^x + 2$. [no GDC]

(a) Find $f^{-1}(x)$. [2]

(b) Write down the domain of f^{-1}. [1]

PROBLEMS

●●● **Question 4.** Let $f(x) = ax + b$, where $a > 0$. It is given that $(f \circ f)(x) = 4x + 9$. [no GDC]

(a) Find the values of a and b. [4]

●●● **Question 5.** Let $f(x) = \dfrac{2x+5}{x-2}$, for $x \neq 2$. [no GDC]

(a) Show that f is its own inverse. [3]

(b) Hence write down $(f \circ f)(7)$. [1]

ANSWERS

1 $f(8) = 16$

2 $g(6) = 11$

3 $x + 7$

4 $\dfrac{x}{4}$

5 $\sqrt[3]{x-1}$

6 2

Q1 (a) $(2x-3)^2 + 1 = 4x^2 - 12x + 10$ (b) $4x^2 - 12x = 0$: $x = 0$ or $x = 3$

Q2 (a) $x(y+1) = 3y - 2$: $f^{-1}(x) = \dfrac{x+2}{3-x}$ (b) $x \neq 3$

Q3 (a) $\ln(x-2)$ (b) $x > 2$

Q4 $a(ax+b) + b = a^2x + ab + b$: $a^2 = 4$, $a = 2$; $3b = 9$, $b = 3$

Q5 (a) $x(y-2) = 2y + 5$ gives $y(x-2) = 2x + 5$, so $f^{-1}(x) = \dfrac{2x+5}{x-2} = f(x)$ (b) 7

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