Exponential and logarithmic functions

Skills sheet · SL AA 2.9 · 2.10

KEY FACTS

a^x: through $(0, 1)$, asymptote $y = 0$, range $y > 0$ D1

$k\,a^x + c$: asymptote $y = c$, y-intercept $k + c$ P1

$y = \log_a x \iff a^y = x$; ln undoes e D2

$\log_a x$: through $(1, 0)$, asymptote $x = 0$, domain $x > 0$ P2

The asymptote moves with the graph W1.

Inside a logarithm: positive W2.

Exact means ln, *not decimals* W2.

WHAT UNLOCKS

- $a^x = b \;\to\; x = \log_a b$; $e^{kx} = b \;\to\; kx = \ln b$.
- a model $A\,b^t \;\to\; A$ at $t = 0$; $b < 1$: decay of $(1 - b) \times 100\,\%$ a step.
- "find k" $\to$ one known value, then ln.
- the inverse of e or ln $\to$ exchange x and y, then the other function.

WORKED EXAMPLE ①

Solve $3 \times 2^x = 48$. no GDC

$2^x = 16$ (M1)

$x = 4$ A1

Method

An exponential equation

① Isolate the power.
② Same base, or take log.
③ Solve for x.

WORKED EXAMPLE ②

Find the inverse of $f(x) = \ln(x - 1) + 2$. no GDC

$x = \ln(y - 1) + 2 \Rightarrow y - 1 = e^{x-2}$ (M1)

$f^{-1}(x) = e^{x-2} + 1$ A1

Method

ln *and* e *undo each other*

① Isolate the ln.
② e to both sides.
③ Isolate y.

The asymptote $x = 1$ ***becomes*** $y = 1$.

FLUENCY

●○○ **1.** The asymptote of $4e^x - 3$ no GDC

●○○ **2.** The y-intercept of $2^x + 5$ no GDC

●○○ **3.** The domain of $\ln(3 - x)$ no GDC

●○○ **4.** The asymptote of $\log_2(x + 5)$ no GDC

●○○ **5.** Solve $e^x = 7$ no GDC

●○○ **6.** The range of $-e^x$ no GDC

REASONING

●●● **Question 1.** Let $f(x) = 4 - e^{2x}$. [no GDC]

(a) Write down the y-intercept and the horizontal asymptote. [2]

(b) Find the exact x-intercept. [2]

(c) Write down the range of f. [1]

●●● **Question 2.** Let $f(x) = \ln(x+3) - 1$, for $x > -3$. [no GDC]

(a) Find the exact x-intercept. [2]

(b) Find $f^{-1}(x)$. [2]

●●● **Question 3.** A population is modelled by $P(t) = 80 \times 1.05^t$, t in years. [with GDC]

(a) Find $P(10)$. [1]

(b) Find the time the population takes to double. [2]

PROBLEMS

●●● **Question 4.** The graph of $f(x) = a \times b^x$, with $b > 0$, passes through $(0, 3)$ and $(2, 12)$. [no GDC]

(a) Find a and b. [3]

●●● **Question 5.** The graph of $f(x) = \ln(x - k)$ meets the x-axis at $x = 5$. [no GDC]

(a) Find k, and write down the vertical asymptote. [3]

ANSWERS

1 $y = -3$

2 6

3 $x < 3$

4 $x = -5$

5 $x = \ln 7$

6 $y < 0$

Q1 (a) 3; $y = 4$ (b) $e^{2x} = 4$: $x = \frac{1}{2}\ln 4 = \ln 2$ (c) $f(x) < 4$

Q2 (a) $\ln(x+3) = 1$: $x = e - 3$ (b) $e^{x+1} - 3$

Q3 (a) 130 (130.311…) (b) $1.05^t = 2$: $t = 14.2$ years (14.2067…)

Q4 $a = 3$; $3b^2 = 12$, $b = 2$

Q5 $\ln(5-k) = 0$: $k = 4$; asymptote $x = 4$

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