Lines, functions and graphs

SL AA · Functions · 2.1 · 2.2 · 2.3 · 2.4 · 2.10

A line is fixed by a point and a gradient; a function by its rule and its domain. The exam reads both from a few numbers: a gradient, an intercept, the values a function cannot take, the values it reaches, the points where two graphs meet.

I STRAIGHT LINES

SL 2.1 · gradient, the three forms, parallel and perpendicular

I.1 The gradient

Definition 1 *gradient*

The **gradient** of the line through $A(x_1, y_1)$ and $B(x_2, y_2)$, with $x_1 \neq x_2$, is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$: the rise for one step across. A horizontal line has gradient 0; a vertical line, $x = k$, has no gradient.

Example 1

Find the gradient of the line through $(-2, 5)$ and $(4, -7)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Label the two points	$(x_1, y_1) = (-2, 5),\ (x_2, y_2) = (4, -7)$
Step 2 Rise over run Same order on top and below.	$m = \dfrac{-7 - 5}{4 - (-2)} = \dfrac{-12}{6}$
Step 3 Simplify	$m = -2$

APPLY

1 The gradient of the line through $(1, 3)$ and $(5, 11)$.

2 The gradient of the line through $(-3, 4)$ and $(3, 1)$.

3 The gradient of the line through $(2, -1)$ and $(2, 6)$.

I.2 The equation of a line

Property 1 *three forms of the equation*

gradient-intercept $y = mx + c$ (gradient m, y-intercept c); **point-gradient** $y - y_1 = m(x - x_1)$, through (x_1, y_1); **general** $ax + by + d = 0$, gradient $-\frac{a}{b}$.

Example 2

Find the equation of the line through $(3, -1)$ with gradient 4. Give it in the form $y = mx + c$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the point-gradient form	$y - (-1) = 4(x - 3)$
Step 2 Expand the bracket	$y + 1 = 4x - 12$
Step 3 Make y the subject	$y = 4x - 13$

APPLY

4 Through $(1, 6)$ with gradient -3, in the form $y = mx + c$.

5 Through $(-1, -4)$ and $(1, 2)$, in the form $y = mx + c$.

6 The gradient and the y-intercept of $2x + 5y - 10 = 0$.

I.3 The general form $ax + by + d = 0$

Watch out 1

"In the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$": clear every fraction, $y = -\frac{1}{2}x + 4$ becomes $x + 2y - 8 = 0$. The gradient of $ax + by + d = 0$ is $-\frac{a}{b}$, not a.

Example 3

Write $y = \frac{2}{3}x - 5$ in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Multiply by the denominator	$3y = 2x - 15$
Step 2 Bring everything to one side	$2x - 3y - 15 = 0$

Check: the gradient $-\frac{2}{-3} = \frac{2}{3}$.

APPLY

7 Write $y = -\frac{1}{4}x + 2$ in the form $ax + by + d = 0$, with integers.

8 Write $y = \frac{3}{2}x + \frac{1}{2}$ in the form $ax + by + d = 0$, with integers.

I.4 Parallel and perpendicular lines

Property 2 *parallel and perpendicular*

Gradients m_1 and m_2: **parallel** when $m_1 = m_2$; **perpendicular** when $m_1 m_2 = -1$, that is $m_2 = -\dfrac{1}{m_1}$.

Example 4

The line L has gradient 2 and passes through $P(2, 3)$. Find the equation of the line perpendicular to L through P.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Perpendicular gradient: $-\dfrac{1}{m}$	$m = -\dfrac{1}{2}$
Step 2 Point-gradient form through P	$y - 3 = -\dfrac{1}{2}(x - 2)$
Step 3 Make y the subject	$y = -\dfrac{1}{2}x + 4$

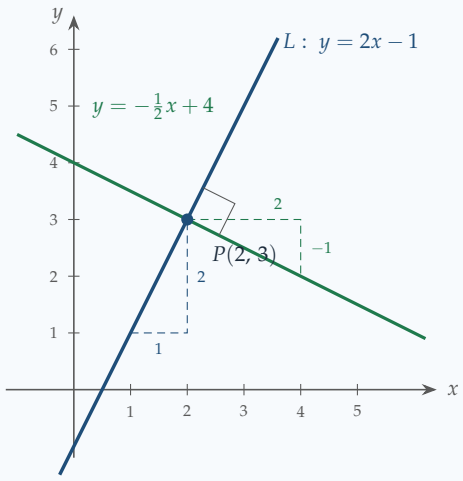


Figure 1 — Example 4 on the graph: L through $P(2, 3)$ with gradient 2, and the perpendicular $y = -\frac{1}{2}x + 4$: "one across, two up" turned through a right angle becomes "two across, one down".

APPLY

9 The line parallel to $y = -2x + 7$ through $(1, 4)$.

10 The line perpendicular to $y = \frac{1}{2}x + 3$ through $(-2, 5)$.

11 Are $2x + y = 4$ and $x - 2y = 6$ perpendicular?

I.5 Where a line meets the axes

Example 5

Find the points where the line $2x + 5y - 20 = 0$ meets the x-axis and the y-axis.

WHAT YOU DO	WHAT YOU WRITE
Step 1 On the x-axis, put $y = 0$	$2x - 20 = 0$: $(10, 0)$
Step 2 On the y-axis, put $x = 0$	$5y - 20 = 0$: $(0, 4)$
Step 3 Write both as points	$(10, 0)$ and $(0, 4)$

APPLY

12 Where does $4x - 3y + 24 = 0$ meet the axes?

13 Where does $y = -2x + 8$ meet the axes?

II FUNCTIONS, DOMAIN AND RANGE

SL 2.2 · 2.4 · the domain, the range, the key features of a graph

II.1 A function: $f(x)$ and $f(x) = k$

Definition 2 *function*

A **function** f gives each x of its **domain** exactly one value $f(x)$. The **range** is the set of the values $f(x)$ takes. On a graph: the domain on the x-axis, the range on the y-axis.

Example 6

Let $f(x) = 3x^2 - 2$. Find $f(-2)$, then solve $f(x) = 25$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Replace x by -2 $(-2)^2 = 4$: the bracket keeps the sign.	$f(-2) = 3(-2)^2 - 2 = 10$
Step 2 Write the equation	$3x^2 - 2 = 25$
Step 3 Solve	$x^2 = 9$: $x = 3$ or $x = -3$

APPLY

1 $g(x) = 5 - 2x$: find $g(4)$, then solve $g(x) = 11$.

2 $h(x) = x^2 + 1$: find $h(-3)$, then solve $h(x) = 17$.

II.2 The largest possible domain

Property 3 *the largest possible domain*

Remove what cannot be computed: no division by 0, no square root of a negative number, no ln of a number ≤ 0.

Example 7

Find the largest possible domain of $f(x) = \sqrt{12 - 3x}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Spot what can fail	a square root: what is under it must be ≥ 0
Step 2 Write the condition	$12 - 3x \geq 0$
Step 3 Solve it Dividing by -3 turns the sign round.	$x \leq 4$

APPLY

3 Largest domain of $\dfrac{1}{2x - 5}$.

4 Largest domain of $\ln(x + 6)$.

5 Largest domain of $\sqrt{x + 1} + \dfrac{1}{x - 3}$.

II.3 The range on a restricted domain

Watch out 2

The range is not always between $f(a)$ and $f(b)$: a vertex, a maximum or a minimum inside the domain gives the least or the greatest value.

Example 8

Find the range of $f(x) = x^2 + 2x - 3$, for $-3 \leq x \leq 2$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Find the vertex	$x = -\dfrac{2}{2} = -1,\ f(-1) = -4$
Step 2 Compute the ends	$f(-3) = 0,\ f(2) = 5$
Step 3 Least and greatest of these The least value is at the vertex, not at an end.	$-4 \leq f(x) \leq 5$

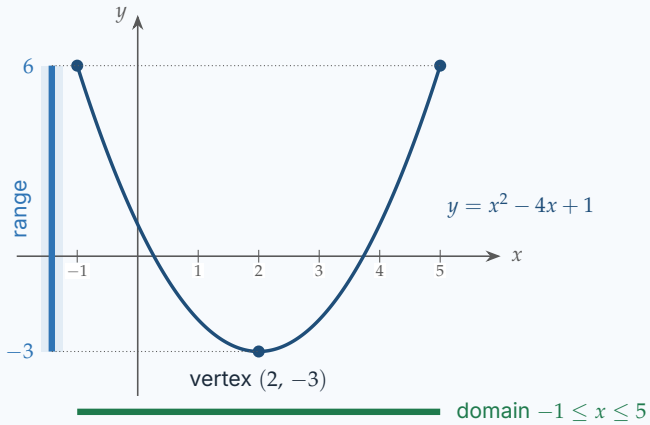


Figure 2 — Exam-style question 4 on the graph: $f(x) = x^2 - 4x + 1$ on $-1 \leq x \leq 5$, range from the vertex -3 up to $f(-1) = f(5) = 6$.

APPLY

6 Range of $f(x) = 4 - x$, for $0 \leq x \leq 6$.

7 Range of $f(x) = x^2 - 8x + 10$, for $0 \leq x \leq 5$.

8 Range of $f(x) = x^2$, for $-2 \leq x \leq 3$.

II.4 Domain and range of a square root

Example 9

Let $f(x) = \sqrt{3x-9} + 4$, for $x \geq 3$. Find the range of f, then solve $f(x) = 7$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The root is never negative	$\sqrt{3x-9} \geq 0$
Step 2 Add the constant	$f(x) \geq 4$
Step 3 Isolate the root, then square	$\sqrt{3x-9} = 3$: $3x - 9 = 9$
Step 4 Solve	$x = 6$

APPLY

9 Range of $f(x) = \sqrt{x-4} - 1$, for $x \geq 4$.

10 Domain and range of $f(x) = 2 + \sqrt{5-x}$.

11 Solve $\sqrt{x+2} = 3$.

II.5 Key features of a graph

Property 4 *key features of a graph*

The y-intercept $f(0)$; the **zeros**, where $f(x) = 0$; the vertex, local maximum and minimum points; the lines of symmetry; the vertical and horizontal **asymptotes**. A sketch shows each with its coordinates.

Example 10

For $f(x) = x^2 - 2x - 8$, write down the y-intercept, the zeros and the vertex of the graph.

WHAT YOU DO	WHAT YOU WRITE
Step 1 y-intercept: $f(0)$	$(0, -8)$
Step 2 Zeros: factorise $f(x) = 0$	$(x-4)(x+2) = 0$: $x = 4, x = -2$
Step 3 Vertex halfway between the zeros	$x = 1$, $f(1) = -9$: $(1, -9)$

APPLY

12 $y = x^2 - 9$: y-intercept and zeros.

13 $y = \dfrac{2}{x+1}$: the vertical asymptote and the y-intercept.

III SOLVING EQUATIONS WITH GRAPHS

SL 2.3 · 2.10 · zeros and intersections, by hand and on the GDC

III.1 The zeros on the GDC

Property 5 *one equation, two graphs*

The solutions of $f(x) = 0$ are the **zeros** of f, where its graph meets the x-axis. The solutions of $f(x) = g(x)$ are the x-coordinates of the points where $y = f(x)$ and $y = g(x)$ meet. On Paper 2: on the GDC.

Example 11

Solve $x^3 - 4x + 2 = 0$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Graph $f(x) = x^3 - 4x + 2$	three crossings of the x-axis
Step 2 Find each zero on the GDC	$x = -2.21,\ x = 0.539,\ x = 1.68$
Step 3 Write the equation and the answers, 3 s.f.	$x^3 - 4x + 2 = 0$: $x = -2.21$, 0.539 or 1.68

APPLY

1 Solve $x^3 - 2x^2 - 1 = 0$.

2 Solve $4 - x^2 = 0$, then check on the GDC.

III.2 Where two graphs meet

Watch out 3

Write the equation you solved, not only the numbers: "$\ln x = 3 - x$", then $x = 2.21$. Give every answer to 3 significant figures, and every solution asked for.

Example 12

Solve $\ln x = 3 - x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 One function for each side	$f(x) = \ln x,\ g(x) = 3 - x$
Step 2 Intersection on the GDC	one point, $x = 2.20794\ldots$
Step 3 Write the answer, 3 s.f.	$x = 2.21$

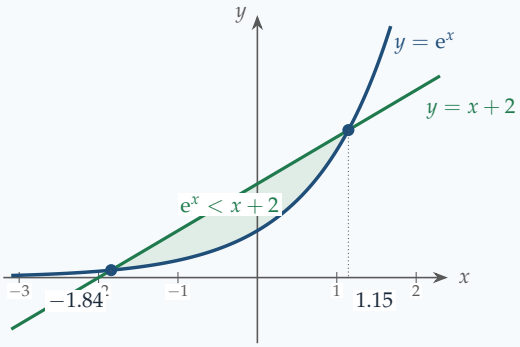


Figure 3 — Exam-style question 5 on the graph: $e^x = x + 2$ has no algebraic solution, but the two graphs meet twice, $x \approx -1.84$ and $x \approx 1.15$.

APPLY

3 Solve $2^x = 5 - x$.

4 Solve $x^2 = e^x$.

5 Solve $4 - x^2 = e^x$.

III.3 An inequality from two graphs

Example 13

Solve $e^{0.5x} > 3 - x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Graph both sides	$y = e^{0.5x}$ and $y = 3 - x$
Step 2 Find where they meet	$x = 1.18841\ldots$
Step 3 Read where the curve is above the line	$x > 1.19$

APPLY

6 Solve $2^x < 5 - x$.

7 Solve $4 - x^2 > e^x$.

IV EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Straight lines

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 7]

The points $A(-1, 2)$ and $B(3, 10)$ lie on the line L_1.

(a) Find the gradient of L_1. [2]

(b) Find the equation of L_1. Give your answer in the form $y = mx + c$. [2]

The line L_2 is perpendicular to L_1 and passes through B.

(c) Find the equation of L_2. Give your answer in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Rise over run	$m = \dfrac{10-2}{3-(-1)}$	(M1)
Step 2 Simplify	$= 2$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Point-gradient form through A	$y - 2 = 2(x+1)$	(M1)
	Either point may be used.	
Step 2 Make y the subject	$y = 2x + 4$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Perpendicular gradient: $-\dfrac{1}{m}$	gradient of L_2: $-\dfrac{1}{2}$	(A1)
	$m_1 m_2 = -1$.	
Step 2 Point-gradient form through B	$y - 10 = -\dfrac{1}{2}(x-3)$	M1
	Through B, with their perpendicular gradient.	
Step 3 Clear the fractions, all on one side	$x + 2y - 23 = 0$	A1
	Any integer multiple, such as $-2x - 4y + 46 = 0$.	

EXAM-STYLE QUESTION 2

Paper 1 · no calculator · [Maximum mark: 6]

The line L has equation $3x - 4y + 12 = 0$.

(a) Find the gradient of L. [2]

(b) Write down the coordinates of the points where L meets the x-axis and the y-axis. [2]

(c) Find the area of the triangle enclosed by L and the two axes. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Make y the subject	$4y = 3x + 12$ *Make y the subject first.*	(M1)
Step 2 Read the gradient	$m = \frac{3}{4}$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 x-axis: put $y = 0$	$(-4, 0)$ *Put $y = 0$.*	A1
Step 2 y-axis: put $x = 0$	$(0, 3)$ *Put $x = 0$.*	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Half base times height, lengths positive	$\frac{1}{2} \times 4 \times 3$ *The lengths are 4 and 3: a length is positive.*	(M1)
Step 2 Compute	$= 6$	A1

B Domain and range

EXAM-STYLE QUESTION 3 — Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = \sqrt{2x-6} + 1$, for $x \geq a$, where a is the smallest value for which f is defined.

(a) Find the value of a. [2]

(b) Find the range of f. [2]

(c) Find the value of x for which $f(x) = 5$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Under the root: ≥ 0	$2x - 6 \geq 0$	(M1)
	The square root needs a number that is not negative.	
Step 2 Solve	$a = 3$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The root is never negative	$\sqrt{2x-6} \geq 0$	(M1)
Step 2 Add the constant	$f(x) \geq 1$	A1
	Or $y \geq 1$, or $[1, \infty[$.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Isolate the root, then square	$\sqrt{2x-6} = 4 \Rightarrow 2x - 6 = 16$	(M1)
Step 2 Solve	$x = 11$	A1

EXAM-STYLE QUESTION 4

Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = x^2 - 4x + 1$, for $-1 \leq x \leq 5$.

(a) Write down the y-intercept of the graph of f. [1]

(b) Find the coordinates of the vertex of the graph of f. [2]

(c) Find the range of f. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 y-intercept: $f(0)$	1 *$f(0) = 1$.*	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Axis: $x = -\dfrac{b}{2a}$	$x = -\dfrac{-4}{2 \times 1} = 2$	(M1)
Step 2 Substitute to find y	$(2, -3)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Compute the ends of the domain	$f(-1) = 6,\ f(5) = 6$ *The ends of the domain.*	(A1)
Step 2 Least and greatest values	$-3 \leq f(x) \leq 6$ *The vertex gives the least value.*	A1

C With the GDC (Paper 2)

EXAM-STYLE QUESTION 5

Paper 2 · calculator · [Maximum mark: 5]

Let $f(x) = e^x$ and $g(x) = x + 2$.

(a) Find the x-coordinates of the points where the graphs of f and g intersect. [2]

(b) Hence solve the inequality $e^x < x + 2$. [2]

(c) Write down the range of f. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Intersections on the GDC	$x = -1.84 \ \ (-1.84140\ldots)$	A1
	$x = 1.15 \ \ (1.14619\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Read where the line is above the curve	the line is above the curve between the intersections	(M1)
Step 2 Write the interval	$-1.84 < x < 1.15$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Read the range from the graph	$f(x) > 0$	A1

The curve never reaches the x-axis: 0 is not in the range.

EXAM-STYLE QUESTION 6

Paper 2 · calculator · [Maximum mark: 6]

Let $f(x) = x^3 - 3x - 1$, for $-2 \le x \le 2$.

(a) Find the zeros of f. [3]

(b) Find the range of f. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Zeros on the GDC	$x = -1.53 \ \ (-1.53208\ldots)$	A1
	$x = -0.347 \ \ (-0.347296\ldots)$	A1
	$x = 1.88 \ \ (1.87938\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Find the extrema	local maximum $(-1, 1)$, local minimum $(1, -3)$	(M1)
	From the GDC, or from $f'(x) = 3x^2 - 3 = 0$.	
Step 2 Compute the ends of the domain	$f(-2) = -3, \ f(2) = 1$	(A1)
	Check the ends: here they equal the extrema.	
Step 3 Least and greatest values	$-3 \le f(x) \le 1$	A1

V WHICH METHOD? READ THE QUESTION

2.10 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"find the gradient" from two points	$\frac{y_2 - y_1}{x_2 - x_1}$	rise over run, signs kept D1
"find the equation of the line"	a point and the gradient	$y - y_1 = m(x - x_1)$, then the form asked P1
"perpendicular"	$m_1 m_2 = -1$	the negative reciprocal, $-\frac{1}{m}$ P2
"a, b, $d \in \mathbb{Z}$"	the general form	clear the fractions: $ax + by + d = 0$ W1
"find the domain"	$\sqrt{}$, $\frac{1}{-}$, ln	what is under the root ≥ 0, the denominator $\neq 0$, inside ln > 0 P3
"find the range"	the graph on its domain	the least and greatest values: vertex, extrema and ends D2
"solve" an equation with e, ln, sin *and* x	Paper 2	graph both sides; intersections on the GDC P5

In the exam 1 *command terms*

Write down: no working needed. **Find**: show the working. **Hence**: use the previous answer. **Sketch**: the shape, with the intercepts, the vertex and the asymptotes labelled. **Domain**: the x-values allowed. **Range**: the y-values reached.

VI ON THE GDC: PAPER 2

SL 2.1–2.4 · 2.10 · NumWorks · every calculator move of this topic

1 The zeros of a function

Grapher → f(x) → Find → Zeros

Type f, draw it, then each zero in turn. **On paper:** "$x^3 - 3x - 1 = 0$", then $x = -1.53$, $x = -0.347$, $x = 1.88$.

GRAPHER · ZEROS

```
f(x)=x^3-3x-1
x=-1.532089 y=0
x=-0.3472964 y=0
x=1.879385 y=0
```

2 Where two graphs meet

Grapher → f(x), g(x) → Find → Intersection

Both sides of the equation as two functions, then each intersection. **On paper:** "$e^x = x + 2$", then $x = -1.84$ or $x = 1.15$.

GRAPHER · INTERSECTION

```
f(x)=e^x g(x)=x+2
x=-1.841406 y=0.1585943
x=1.146193 y=3.146193
```

3 The range on a domain

Grapher → Axes → the domain → Maximum, Minimum

Set the domain in the axes, then the extrema; compare them with the values at the ends. **On paper:** the extrema and the ends, then $-3 \le f(x) \le 1$.

GRAPHER · MAXIMUM

```
f(x)=x^3-3x-1, -2<=x<=2
Maximum: x=-1 y=1
Minimum: x=1 y=-3
f(-2)=-3 f(2)=1
```

ANSWERS TO THE APPLY QUESTIONS

I.1 2

I.2 $-\frac{1}{2}$

I.3 none: the line $x = 2$ is vertical

I.4 $y = -3x + 9$

I.5 $m = 3;\ y = 3x - 1$

I.6 $-\frac{2}{5};\ 2$

I.7 $x + 4y - 8 = 0$

I.8 $3x - 2y + 1 = 0$

I.9 $y = -2x + 6$

I.10 $y = -2x + 1$

I.11 yes: $(-2) \times \frac{1}{2} = -1$

I.12 $(-6,\ 0)$ and $(0,\ 8)$

I.13 $(4,\ 0)$ and $(0,\ 8)$

II.1 $-3;\ x = -3$

II.2 $10;\ x = \pm 4$

II.3 $x \neq \frac{5}{2}$

II.4 $x > -6$

II.5 $x \ge -1,\ x \neq 3$

II.6 $-2 \le f(x) \le 4$

II.7 $-6 \le f(x) \le 10$

II.8 $0 \le f(x) \le 9$

II.9 $f(x) \ge -1$

II.10 $x \le 5;\ f(x) \ge 2$

II.11 $x = 7$

II.12 $(0,\ -9);\ x = \pm 3$

II.13 $x = -1;\ (0,\ 2)$

III.1 $x = 2.21$ (one zero)

III.2 $x = \pm 2$

III.3 $x = 1.72$

III.4 $x = -0.703$

III.5 $x = -1.96$ or $x = 1.06$

III.6 $x < 1.72$

III.7 $-1.96 < x < 1.06$

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