Lines, functions and graphs

Skills sheet · SL AA 2.1 · 2.2 · 2.3 · 2.4 · 2.10

KEY FACTS

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ D1

$y - y_1 = m(x - x_1)$; $ax + by + d = 0$ has gradient $-\frac{a}{b}$ P1

Parallel: $m_1 = m_2$; perpendicular: $m_1 m_2 = -1$ P2

Domain: $\sqrt{u}$ needs $u \geq 0$, $\frac{1}{u}$ needs $u \neq 0$, $\ln u$ needs $u > 0$ P3

Integers: clear the fractions W1.

The range: vertex and ends, not ends alone W2.

Write the equation you solved W3.

WHAT UNLOCKS

- "perpendicular" → the negative reciprocal of the gradient.
- "largest possible domain" → what is under a root, in a denominator, inside a logarithm.
- "range" → a sketch on the domain: least and greatest values.
- an equation with e, ln or sin and x → two graphs, intersections on the GDC.

WORKED EXAMPLE ①

Find the line through $(1, -2)$ perpendicular to $y = 3x + 5$, in the form $ax + by + d = 0$. no GDC

gradient $-\dfrac{1}{3}$ (A1)

$y + 2 = -\dfrac{1}{3}(x - 1)$ M1

$x + 3y + 5 = 0$ A1

Method

How do I find a line?

① Its gradient: given, two points, $-\frac{1}{m}$.
② One point on it.
③ $y - y_1 = m(x - x_1)$.
④ The form asked for.

WORKED EXAMPLE ②

Find the domain and the range of $f(x) = 2 + \sqrt{x - 1}$. no GDC

$x - 1 \geq 0$: $x \geq 1$ (M1)A1

$\sqrt{x - 1} \geq 0$, so $f(x) \geq 2$ (M1)A1

Method

Domain, then range

① What cannot be computed?
② Exclude it: the domain.
③ Sketch on the domain.
④ Lowest and highest: the range.

The range is read on the y-axis.

FLUENCY

1. The gradient through $(-2, 1)$ and $(4, 10)$ no GDC

2. The line of gradient -3 through $(2, 5)$ no GDC

3. The gradient of $5x - 2y + 8 = 0$ no GDC

4. The domain of $\sqrt{4 - x}$ no GDC

5. The domain of $\dfrac{3}{2x - 1}$ no GDC

6. The range of $x^2 + 3$, $x \in \mathbb{R}$ no GDC

REASONING

●●● **Question 1.** The points $P(-3, 4)$ and $Q(5, 0)$ are given. [no GDC]

(a) Find the equation of the line PQ, in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [3]

(b) Find the equation of the line through $R(1, 7)$ parallel to PQ. [2]

●●● **Question 2.** Let $f(x) = x^2 - 6x + 5$, for $0 \leq x \leq 7$. [no GDC]

(a) Find the coordinates of the vertex of the graph of f. [2]

(b) Find the range of f. [2]

●●● **Question 3.** Let $f(x) = \ln(2x - 6)$. [no GDC]

(a) Find the largest possible domain of f. [2]

(b) Solve $f(x) = 0$. [2]

PROBLEMS

●●● **Question 4.** The points $A(1, 1)$, $B(5, 3)$ and $C(k, 7)$ are such that $[AC]$ is perpendicular to $[AB]$. [no GDC]

(a) Find the value of k. [3]

●●● **Question 5.** The lines $2x + ky = 6$ and $(k - 1)x + 3y = 4$ are parallel. [no GDC]

(a) Find the possible values of k. [4]

ANSWERS

1 $\dfrac{9}{6} = \dfrac{3}{2}$

2 $y = -3x + 11$

3 $\dfrac{5}{2}$

4 $x \leq 4$

5 $x \neq \dfrac{1}{2}$

6 $f(x) \geq 3$

Q1 (a) $m = -\frac{1}{2}$: $x + 2y - 5 = 0$ (b) $y = -\frac{1}{2}x + \frac{15}{2}$, or $x + 2y - 15 = 0$

Q2 (a) $(3, -4)$ (b) $f(0) = 5$, $f(7) = 12$: $-4 \leq f(x) \leq 12$

Q3 (a) $2x - 6 > 0$: $x > 3$ (b) $2x - 6 = 1$: $x = \frac{7}{2}$

Q4 $m_{AB} = \frac{1}{2}$, so $m_{AC} = \dfrac{6}{k-1} = -2$: $k = -2$

Q5 $-\dfrac{2}{k} = -\dfrac{k-1}{3}$: $k^2 - k - 6 = 0$, $k = 3$ or $k = -2$ (the lines are then distinct)

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