Quadratic functions

Skills sheet · SL AA 2.6 · 2.7

KEY FACTS

$a(x-h)^2+k$: vertex (h, k); $a(x-p)(x-q)$: zeros p, q P1

Axis of symmetry: $x = -\dfrac{b}{2a}$ P1

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ P3

$\Delta > 0$: two roots; $\Delta = 0$: equal; $\Delta < 0$: none D2

$(x+3)^2$: the vertex is at $x=-3$ W1.
"real roots" means $\Delta \geq 0$ W2.
$x^2 > 9$: two pieces W3.

WHAT UNLOCKS

- "the vertex", "the maximum" $\rightarrow$ complete the square, or $x = -\frac{b}{2a}$.
- "touches", "tangent", "equal roots" $\rightarrow$ $\Delta = 0$.
- "never meets", "no real roots" $\rightarrow$ $\Delta < 0$, then an inequality.
- an inequality $\rightarrow$ the roots, then a sketch.

WORKED EXAMPLE ①

Write x^2-8x+3 in the form $(x-h)^2+k$. no GDC

$(x-4)^2-16+3$ (M1)

$(x-4)^2-13$ A1

Method
Completing the square
① Take a out of the x terms.
② Halve b: $(x+\frac{b}{2})^2$.
③ Subtract $\left(\frac{b}{2}\right)^2$.
④ Collect the constants.

WORKED EXAMPLE ②

Solve $x^2-2x-8>0$. no GDC

$(x+2)(x-4)=0$: $x=-2, x=4$ (M1)A1

$x<-2$ or $x>4$ A1

Method
A quadratic inequality
① Solve the equation.
② Sketch the parabola.
③ Read above or below the axis.
"Outside the roots" is two inequalities.

FLUENCY

1. The vertex of $(x-5)^2+2$ no GDC

2. The axis of symmetry of $3x^2-12x+1$ no GDC

3. The zeros of $2(x+1)(x-4)$ no GDC

4. Δ for x^2+3x+5: how many roots? no GDC

5. Solve $x^2=4x$ no GDC

6. Solve $x^2<9$ no GDC

REASONING

●●● **Question 1.** A parabola has zeros 1 and 5, and its vertex is $(3, -8)$. [no GDC]

(a) Find its equation in the form $a(x-p)(x-q)$. [3]

(b) Write it in the form ax^2+bx+c. [1]

●●● **Question 2.** Consider the inequality $2x^2-5x-3 \leq 0$. [no GDC]

(a) Solve the inequality. [3]

●●● **Question 3.** The equation $x^2+kx+k+3=0$ has two equal roots. [no GDC]

(a) Find the possible values of k. [4]

PROBLEMS

●●● **Question 4.** The line $y=2x+c$ is a tangent to the curve $y=x^2$. [no GDC]

(a) Find c and the point of contact. [4]

●●● **Question 5.** The line $y=mx$ meets the curve $y=x^2+4$ at two distinct points. [no GDC]

(a) Find the possible values of m. [4]

ANSWERS

1 $(5,\,2)$

2 $x = 2$

3 -1 and 4

4 $9 - 20 = -11 < 0$: no real roots

5 $x = 0$ or $x = 4$

6 $-3 < x < 3$

Q1 (a) $a(3-1)(3-5) = -8$: $a = 2$, $2(x-1)(x-5)$ (b) $2x^2 - 12x + 10$

Q2 $(2x+1)(x-3)$: $-\frac{1}{2} \le x \le 3$

Q3 $k^2 - 4(k+3) = 0$: $k^2 - 4k - 12 = 0$, $k = 6$ or $k = -2$

Q4 $x^2 - 2x - c = 0$, $\Delta = 4 + 4c = 0$: $c = -1$; point $(1,\,1)$

Q5 $x^2 - mx + 4 = 0$, $m^2 - 16 > 0$: $m < -4$ or $m > 4$

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