Rational functions

Skills sheet · SL AA 2.8

KEY FACTS

$\frac{1}{x}$: asymptotes $x = 0$, $y = 0$; it is self-inverse D1

$\frac{ax+b}{cx+d}$: $x = -\frac{d}{c}$ and $y = \frac{a}{c}$ P2

Intercepts $\left(-\frac{b}{a}, 0\right)$ and $\left(0, \frac{b}{d}\right)$ P2

Domain $x \neq -\frac{d}{c}$, range $y \neq \frac{a}{c}$ P3

0 is not a value of $\frac{1}{x}$ W1.

The horizontal asymptote is $\frac{a}{c}$, not $\frac{b}{d}$ W2.

Asymptotes as equations, dashed W2.

WHAT UNLOCKS

- "asymptotes" → the denominator, then the ratio of the x-terms.
- "range" → every value but the horizontal asymptote.
- asymptotes given → read the coefficients from them.
- $f(x) > g(x)$ → intersections, and the vertical asymptote splits the answer.

WORKED EXAMPLE ①

Write down the asymptotes of $y = \dfrac{5x+1}{2x-4}$. no GDC

$2x - 4 = 0$: $x = 2$ A1

$y = \dfrac{5}{2}$ A1

Method

Asymptotes of $\frac{ax+b}{cx+d}$

① Vertical: denominator $= 0$.
② Horizontal: $\frac{a}{c}$.
③ Write both as equations.

WORKED EXAMPLE ②

Write $2 + \dfrac{3}{x+1}$ as a single fraction. no GDC

$\dfrac{2(x+1)+3}{x+1}$ (M1)

$= \dfrac{2x+5}{x+1}$ A1

Method

One fraction

① The common denominator.
② Expand the numerator.
③ Read: asymptotes $x = -1$, $y = 2$.

FLUENCY

1. The vertical asymptote of $\dfrac{1}{x-4}$ no GDC

2. The horizontal asymptote of $\dfrac{6x+1}{3x-2}$ no GDC

3. The x-intercept of $\dfrac{2x-5}{x+1}$ no GDC

4. The y-intercept of $\dfrac{x+6}{x-3}$ no GDC

5. The domain of $\dfrac{7}{2x+1}$ no GDC

6. The range of $\dfrac{1}{x} + 4$ no GDC

REASONING

●●● **Question 1.** Let $f(x) = \dfrac{x-3}{x+2}$, for $x \neq -2$. [no GDC]

(a) Write down the equations of the asymptotes. [2]

(b) Find the intercepts with the axes. [2]

●●● **Question 2.** Let $f(x) = \dfrac{2x+3}{x-1}$, for $x \neq 1$. [no GDC]

(a) Find $f^{-1}(x)$. [3]

(b) Solve $f(x) = 5$. [2]

PROBLEMS

●●● **Question 3.** Let $f(x) = \dfrac{ax+b}{x-2}$. The graph of f has the horizontal asymptote $y = 3$ and passes through $(0, -1)$. [no GDC]

(a) Find a and b. [3]

●●● **Question 4.** Let $f(x) = \dfrac{3x+1}{x-1}$, for $x \neq 1$. [no GDC]

(a) Find the value of k for which the equation $f(x) = k$ has no solution. [2]

ANSWERS

1 $x = 4$

2 $y = 2$

3 $x = \frac{5}{2}$

4 -2

5 $x \neq -\frac{1}{2}$

6 $y \neq 4$

Q1 (a) $x = -2, y = 1$ (b) $(3, 0)$ and $\left(0, -\frac{3}{2}\right)$

Q2 (a) $y(x-2) = x+3$: $f^{-1}(x) = \dfrac{x+3}{x-2}$ (b) $2x+3 = 5x-5$: $x = \frac{8}{3}$

Q3 $a = 3$; $\frac{b}{-2} = -1$: $b = 2$

Q4 $k = 3$, the horizontal asymptote: $\frac{3x+1}{x-1} = 3$ gives $1 = -3$, impossible

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