Transformations of graphs

Change the formula a little and the graph moves: it slides, flips or stretches. What happens outside the bracket acts on the y-coordinates, as we expect; what happens inside acts on the x-coordinates, the other way round.

I TRANSLATIONS AND REFLECTIONS

SL 2.11 · $f(x)+k$, $f(x-h)$, $-f(x)$, $f(-x)$

I.1 Up and down: $f(x)+k$

Property 1 *translations*

$y = f(x)+k$: the graph moves k units **up** (down if $k < 0$). $y = f(x-h)$: it moves h units to the **right** (left if $h < 0$). $y = f(x-h)+k$: translation by $\begin{pmatrix} h \\ k \end{pmatrix}$, $(x, y) \to (x+h, y+k)$.

Example 1

The point $(2, 5)$ lies on the graph of $y = f(x)$. Find its image on the graph of $y = f(x) - 3$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Outside the bracket: the y-coordinate	$y \to y - 3$
Step 2 Apply it	$5 - 3 = 2$
Step 3 Write the image	$(2, 2)$

APPLY

1 $(-1, 6)$ is on $y = f(x)$: its image on $y = f(x)+4$.

2 The asymptote of $y = 2^x + 3$.

I.2 Left and right: $f(x-h)$

Watch out 1

Inside the bracket, the sign is the opposite of the move: $f(x-3)$ moves the graph 3 to the *right*, $f(x+3)$ moves it 3 to the left. An asymptote moves with the graph.

Example 2

The point $(2, 5)$ lies on the graph of $y = f(x)$. Find its image on the graph of $y = f(x+4)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Inside the bracket: the x-coordinate	$+4$ inside: 4 to the left
Step 2 Apply it	$2 - 4 = -2$
Step 3 Write the image	$(-2, 5)$

APPLY

3 $(3, -1)$ is on $y = f(x)$: its image on $y = f(x-5)$.

4 The vertical asymptote of $y = \dfrac{1}{x-2}$, from that of $\dfrac{1}{x}$.

I.3 Translation by a vector

Example 3

The graph of $y = x^2$ is translated by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$. Write down the equation of the new graph and its vertex.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $h = -2$ goes inside, with the opposite sign	$(x+2)^2$
Step 2 $k = 5$ goes outside	$y = (x+2)^2 + 5$
Step 3 The vertex moves too	$(0, 0) \to (-2, 5)$

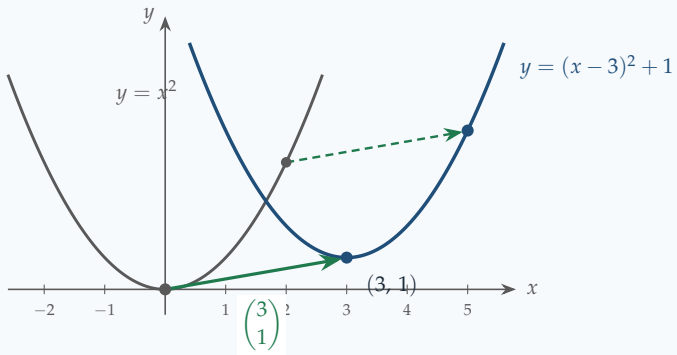


Figure 1 — $y = x^2$ and $y = (x-3)^2 + 1$: every point moves 3 to the right and 1 up, the vertex from $(0, 0)$ to $(3, 1)$: the same move as Example 3, with the vector $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

APPLY

5 $y = x^2$ translated by $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

6 $y = e^x$ translated by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$: equation and asymptote.

I.4 Reflections

Property 2 *reflections*

$y = -f(x)$: reflection in the x-axis, $(x, y) \to (x, -y)$. $y = f(-x)$: reflection in the y-axis, $(x, y) \to (-x, y)$.

Example 4

The point $(4, -3)$ lies on the graph of $y = f(x)$. Find its images on $y = -f(x)$ and on $y = f(-x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $-f(x)$: change the sign of y	$(4, 3)$
Step 2 $f(-x)$: change the sign of x	$(-4, -3)$

APPLY

7 $(-2, 7)$ on $y = f(x)$: images on $-f(x)$ and on $f(-x)$.

8 The equation of $y = e^x$ reflected in the y-axis.

II STRETCHES, AND TRANSFORMATIONS IN SEQUENCE

SL 2.11 · $pf(x)$, $f(qx)$, composite transformations

II.1 Vertical stretch: $p\,f(x)$

Property 3 *stretches*

$y = p\,f(x)$: **vertical stretch**, scale factor p, $(x,\,y) \to (x,\,py)$. $y = f(qx)$: **horizontal stretch**, scale factor $\dfrac{1}{q}$, $(x,\,y) \to \left(\dfrac{x}{q},\,y\right)$.

Example 5

The point $(3,\,4)$ lies on the graph of $y = f(x)$. Find its images on $y = 3f(x)$ and on $y = \frac{1}{2}f(x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Outside: multiply y	$4 \times 3 = 12,\ 4 \times \frac{1}{2} = 2$
Step 2 x does not change	$(3,\,12)$ and $(3,\,2)$

APPLY

1 $(-1,\,5)$ on $y = f(x)$: image on $y = 4f(x)$.

2 f has a zero at $x = 2$. A zero of $3f(x)$?

II.2 Horizontal stretch: $f(qx)$

Watch out 2

$f(2x)$ halves the x-coordinates, scale factor $\frac{1}{2}$: it does not double them.

Example 6

The point $(6,\,4)$ lies on the graph of $y = f(x)$. Find its images on $y = f(2x)$ and on $y = f\left(\frac{x}{3}\right)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Inside: divide x by q	$\dfrac{6}{2} = 3,\ 6 \div \frac{1}{3} = 18$
Step 2 y does not change	$(3,\,4)$ and $(18,\,4)$

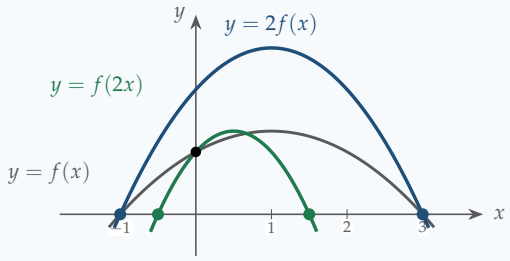


Figure 2 — $y = f(x)$, $y = 2f(x)$, twice as tall, and $y = f(2x)$, half as wide: the vertical stretch keeps the zeros, the horizontal one keeps the y-intercept.

APPLY

3 $(-4,\ 1)$ on $y = f(x)$: image on $y = f(4x)$.

4 $(-4,\ 1)$ on $y = f(x)$: image on $y = f\left(\frac{1}{2}x\right)$.

II.3 Transformations in sequence

Property 4 *transformations in sequence*

$y = af\big(b(x-h)\big) + k$: horizontally, stretch by $\frac{1}{b}$ then translate by h; vertically, stretch by a (negative a also reflects) then translate by k. $(x,\ y) \to \left(\frac{x}{b} + h,\ ay + k\right)$.

Example 7

The point $(2,\ -1)$ lies on the graph of $y = f(x)$. Find its image on $y = 2f(x+3) - 5$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 x: inside, $+3$ is 3 to the left	$2 - 3 = -1$
Step 2 y: stretch first, then translate Multiply before you add.	$2 \times (-1) - 5 = -7$
Step 3 Write the image	$(-1,\ -7)$

APPLY

5 $(4,\ 6)$ on $y = f(x)$: image on $y = -f(x+1) + 2$.

6 $(-2,\ 3)$ on $y = f(x)$: image on $y = 3f(2x)$.

II.4 Describing the transformations

Example 8

Describe the transformations that map $y = x^2$ onto $y = 3(x+1)^2 - 4$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Inside the bracket	translation 1 unit to the left
Step 2 The factor outside	vertical stretch, scale factor 3
Step 3 The constant, last	translation 4 units down

APPLY

7 Describe $y = x^3 \to y = -(x-2)^3$.

8 Describe $y = \sqrt{x} \to y = \sqrt{\frac{x}{2}} + 1$.

II.5 The equation of a transformed graph

Example 9

Let $f(x) = x^2 - 4x$. The graph of g is the graph of f translated by $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find $g(x)$ in the form $x^2 + bx + c$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write g with f	$g(x) = f(x-2) + 1$
Step 2 Replace every x by $x - 2$	$(x-2)^2 - 4(x-2) + 1$
Step 3 Expand and simplify	$g(x) = x^2 - 8x + 13$

APPLY

9 $f(x) = 2x + 1$, $g(x) = f(x + 3) - 4$: find $g(x)$.

10 $y = e^x$ reflected in the x-axis, then 5 up: the equation.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Where a point goes

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 5]

The point $A(3, -2)$ lies on the graph of $y = f(x)$. Write down the coordinates of the image of A on each of the following graphs.

(a) $y = f(x) + 4$ [1]

(b) $y = f(x - 1)$ [1]

(c) $y = -f(x)$ [1]

(d) $y = f(-x)$ [1]

(e) $y = 2f(x)$ [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Outside: add 4 to y	$(3, 2)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Inside: x moves 1 to the right	$(4, -2)$ *Right, not left.*	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Change the sign of y	$(3, 2)$	A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Change the sign of x	$(-3, -2)$	A1

WORKING (e)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Outside: multiply y by 2	$(3, -4)$	A1

B Describe the transformations

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = x^2$ and $g(x) = 2(x-3)^2 + 1$. The graph of g is obtained from the graph of f by three transformations.

(a) Describe these three transformations, in order. [3]

(b) Write down the coordinates of the vertex of the graph of g. [1]

(c) The point $(1, 1)$ lies on the graph of f. Find the coordinates of its image on the graph of g. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Inside the bracket	translation 3 units to the right	A1
	Or by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.	
Step 2 The factor outside	vertical stretch, scale factor 2	A1
Step 3 The constant, last	translation 1 unit up	A1
	Stretch before this translation: up 1 after the stretch.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The vertex $(0, 0)$ moves by the translations	$(3, 1)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 x and y separately	$(1+3,\ 2\times 1+1)$	(M1)
Step 2 Write the image	$(4, 3)$	A1
	Check: $g(4) = 2\times 1+1 = 3$.	

C Key features transformed

EXAM-STYLE QUESTION 3 Paper 1 · no calculator · [Maximum mark: 6]

The graph of $y = f(x)$ has a local maximum at $(2, 5)$ and the horizontal asymptote $y = 1$.

(a) Let $g(x) = 3f(x-1) - 2$. Find the coordinates of the local maximum of the graph of g. [2]

(b) Write down the equation of the horizontal asymptote of the graph of g. [2]

(c) Let $h(x) = f(2x)$. Write down the coordinates of the local maximum of the graph of h. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 x and y separately: stretch, then translate	x: $2 + 1 = 3$; y: $3 \times 5 - 2 = 13$	(M1)
Step 2 Write the image	$(3, 13)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The asymptote moves like the y-coordinates	$y = 3 \times 1 - 2$	(M1)
	An asymptote $y = c$ moves like a y-coordinate.	
Step 2 Write it as an equation	$y = 1$	A1
	Here it happens to stay where it was.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Divide x by q	x: $\dfrac{2}{2} = 1$	(M1)
	Scale factor $\frac{1}{2}$ on x; y unchanged.	
Step 2 Write the image	$(1, 5)$	A1

D With the GDC (Paper 2)

EXAM-STYLE QUESTION 4 Paper 2 · calculator · [Maximum mark: 5]

Let $f(x) = x\,e^{-x}$. The graph of f has a maximum point at $\left(1, e^{-1}\right)$. Let $g(x) = 2f(x-1)$.

(a) Write down the coordinates of the maximum point of the graph of g. [2]

(b) Solve $g(x) = 0.5$. [2]

(c) Describe the two transformations that map the graph of f onto the graph of g. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 x: translate	$x = 1 + 1 = 2$	A1
Step 2 y: stretch	$y = 2e^{-1} = 0.736 \;\; (0.735758\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Intersections on the GDC	$x = 1.36 \;\; (1.35740\ldots)$	A1
	$x = 3.15 \;\; (3.15329\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Describe both transformations	translation 1 unit to the right, and vertical stretch, scale factor 2	A1

Both, in either order: they act on different coordinates.

IV WHICH METHOD? READ THE QUESTION

2.11 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
$y = f(x) + k$	outside: on y	up k; $(x, y) \to (x, y + k)$ P1
$y = f(x - h)$	inside: on x, opposite sign	right h; $(x, y) \to (x + h, y)$ W1
$y = -f(x),\ y = f(-x)$	the sign of y, of x	reflection in the x-axis, in the y-axis P2
$y = p\,f(x)$	outside: on y	vertical stretch, scale factor p P3
$y = f(qx)$	inside: on x, inverse	horizontal stretch, scale factor $\frac{1}{q}$ W2
"describe the transformations", in order	$a\,f\big(b(x - h)\big) + k$	name, direction and size of each: stretches before translations P4
the image of a point, a maximum, an asymptote	$\left(\frac{x}{b} + h,\ ay + k\right)$	apply the same rule to the coordinates P4

In the exam 1 *command terms*

Describe: name each transformation, its direction or axis, its size or scale factor. **Translation**: give the vector, or "3 units to the right". **Stretch**: "vertical" or "horizontal", with the scale factor. **In order**: the order changes the result.

V ON THE GDC: PAPER 2

SL 2.11 · NumWorks · every calculator move of this topic

1 See a transformation

Grapher → f(x), g(x)=f(x−1)+2 → Graph

Type f once, then g in terms of f: the GDC draws the image. **On paper:** name the transformations; the screen only confirms them.

GRAPHER · EXPRESSIONS

```
f(x)=x*e^(-x)
g(x)=2*f(x-1)
two graphs, one scaled copy
```

2 The image of a maximum

Grapher → g(x) → Find → Maximum

Find the maximum of f and of g: $x + 1$, $2y$. **On paper:** $(2,\ 0.736)$, from $(1,\ 0.368)$.

GRAPHER · MAXIMUM

```
f: x=1 y=0.3678794
g: x=2 y=0.7357589
```

ANSWERS TO THE APPLY QUESTIONS

I.1 $(-1, 10)$

I.2 $y = 3$

I.3 $(8, -1)$

I.4 $x = 2$ (2 to the right)

I.5 $y = (x-1)^2 - 3$

I.6 $y = \mathrm{e}^x - 2$; $y = -2$

I.7 $(-2, -7)$; $(2, 7)$

I.8 $y = \mathrm{e}^{-x}$

II.1 $(-1, 20)$

II.2 $x = 2$: zeros stay

II.3 $(-1, 1)$

II.4 $(-8, 1)$

II.5 $(3, -4)$

II.6 $(-1, 9)$

II.7 translation 2 right; reflection in the x-axis

II.8 horizontal stretch, factor 2; translation 1 up

II.9 $2x + 3$

II.10 $y = 5 - \mathrm{e}^x$

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