Transformations of graphs

Skills sheet · SL AA 2.11

KEY FACTS

$f(x)+k$: up k; $f(x-h)$: right h P1
$-f(x)$: in the x-axis; $f(-x)$: in the y-axis P2
$p\,f(x)$: vertical $\times p$; $f(qx)$: horizontal $\times\frac{1}{q}$ P3
$a\,f\big(b(x-h)\big)+k$: $(x,\,y)\to\left(\frac{x}{b}+h,\;ay+k\right)$ P4

Inside the bracket: the opposite W1.
$f(2x)$ halves the x-coordinates W2.
Stretch first, translate after W2.

WHAT UNLOCKS

- outside the bracket $\to$ the y-coordinates, as written.
- inside the bracket $\to$ the x-coordinates, the inverse operation.
- "describe" $\to$ name, direction, size; in order.
- an asymptote $y=c$ $\to$ moves like a y-coordinate; $x=c$ like an x-coordinate.

WORKED EXAMPLE ①

$(4,\,6)$ is on $y=f(x)$. Find its image on $y=\frac{1}{2}f(x)+1$. no GDC

y: $\frac{1}{2}\times 6+1$ (M1)
$(4,\,4)$ A1

Method
The image of a point
① Outside: on y, in the order written.
② Inside: on x, undone.
③ Check with the formula.

WORKED EXAMPLE ②

Describe the transformation from $y=f(x)$ to $y=f(x+1)-2$. no GDC

translation by the vector $\begin{pmatrix}-1\\-2\end{pmatrix}$ A1A1

Method
Describing a transformation
① Its name.
② Its direction or axis.
③ Its size or scale factor.
"Moves" is not a transformation.

FLUENCY

1. Describe $y=f(x)-7$ no GDC
2. Describe $y=f(x+3)$ no GDC
3. Describe $y=-f(x)$ no GDC
4. Describe $y=f(-x)$ no GDC
5. Describe $y=4f(x)$ no GDC
6. Describe $y=f\left(\frac{1}{2}x\right)$ no GDC

REASONING

●●● **Question 1.** The graph of $y = x^2$ is mapped onto the graph of $y = (x-4)^2 + 1$. [no GDC]

(a) Describe the transformation. [2]

(b) Write down the vertex of the image. [1]

●●● **Question 2.** The point $(2, -5)$ lies on the graph of $y = f(x)$. [no GDC]

(a) Find its image on the graph of $y = -f(x-1) + 2$. [2]

●●● **Question 3.** The graph of $y = f(x)$ has asymptotes $x = 1$ and $y = 0$. Let $g(x) = f(2x) + 3$. [no GDC]

(a) Write down the asymptotes of the graph of g. [2]

PROBLEMS

●●● **Question 4.** Let $f(x) = x^2 - 2x$ and $g(x) = f(x+1) + k$. The graph of g touches the x-axis. [no GDC]

(a) Find k. [3]

●●● **Question 5.** The graph of $y = e^x$ is reflected in the y-axis, then translated 2 units up, giving the graph of $y = g(x)$. [no GDC]

(a) Write down $g(x)$ and the range of g. [3]

ANSWERS

1 translation 7 units down

2 translation 3 units to the left

3 reflection in the x-axis

4 reflection in the y-axis

5 vertical stretch, scale factor 4

6 horizontal stretch, scale factor 2

Q1 (a) translation by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ (b) $(4,\ 1)$

Q2 $(2+1,\ -(-5)+2) = (3,\ 7)$

Q3 $x = \frac{1}{2}$ and $y = 3$

Q4 $f(x+1) = (x+1)^2 - 2(x+1) = x^2 - 1$; $g(x) = x^2 - 1 + k$ touches the axis when $k = 1$

Q5 $g(x) = \mathrm{e}^{-x} + 2$; range $g(x) > 2$

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