Compound angle identities

The sine of a sum, $\sin(A+B)$, is not $\sin A + \sin B$. The six compound angle identities say what it is, and everything else in the chapter follows from them: the double angles, the tangent of a sum, and the symmetries that turn sin into cos by a shift of $\frac{\pi}{2}$. All of them are in the formula booklet; the marks are in choosing one and expanding it cleanly.

I SUMS AND DIFFERENCES OF ANGLES

AHL 3.10 · $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$, $\tan 2\theta$

I.1 Sums and differences

Property 1 *the compound angle identities*

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B,$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

In cos the sign changes; in sin it does not.

Example 1

Find the exact value of $\cos \dfrac{\pi}{12}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 A difference of known angles	$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$
Step 2 Expand: the sign changes in cos	$\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$
Step 3 Exact values	$\frac{1}{2}\times\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2} = \dfrac{\sqrt{2}+\sqrt{6}}{4}$

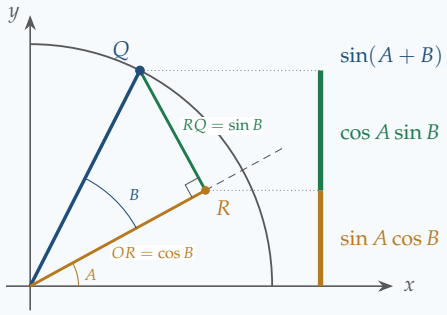


Figure 1 — On the unit circle, the point at angle $A+B$ is reached by turning A, then B. Its height $\sin(A+B)$ is the sum of two pieces, $\sin A \cos B$ and $\cos A \sin B$.

APPLY

1 $\sin\frac{7\pi}{12}$, exactly.

2 A, B acute, $\cos A = \frac{4}{5}$, $\sin B = \frac{5}{13}$: $\cos(A-B)$.

I.2 The tangent of a sum, and double angles

Property 2 *double angles*

With $B = A$: $\sin 2A = 2\sin A\cos A$, $\cos 2A = \cos^2 A - \sin^2 A$, and

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}.$$

An exact value such as $\sin\frac{\pi}{12}$ comes from a difference of known angles, $\frac{\pi}{3} - \frac{\pi}{4}$.

Example 2

A and B are acute, with $\tan A = 4$ and $\tan B = \dfrac{3}{5}$. Find $A - B$ exactly, and $\tan 2A$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $\tan(A-B)$ formula	$\dfrac{4-\frac{3}{5}}{1+4\times\frac{3}{5}} = \dfrac{17/5}{17/5} = 1$
Step 2 Place $A-B$	$-\frac{\pi}{2} < A-B < \frac{\pi}{2}$ and $\tan(A-B) > 0$: $A - B = \frac{\pi}{4}$
Step 3 $\tan 2A$ formula	$\dfrac{2\times 4}{1-16} = -\dfrac{8}{15}$

Watch out 1

To find the angle $A+B$ from $\tan(A+B)$, place $A+B$ first: if A and B are acute, $0 < A+B < \pi$, and a negative tangent means an obtuse angle, $\pi - \arctan|\tan(A+B)|$.

APPLY

3 $\sin A = \frac{3}{5}$, A acute: $\sin 2A$ and $\cos 2A$.

4 $\tan\left(\frac{\pi}{4} + x\right)$ in terms of $t = \tan x$.

II.1 Symmetries

Property 3 *relations between the functions*

Expanding with the compound identities:

$$\sin\left(\tfrac{\pi}{2}-x\right)=\cos x,\quad \cos\left(\tfrac{\pi}{2}-x\right)=\sin x,\quad \tan\left(\tfrac{\pi}{2}-x\right)=\cot x,$$

$$\sin(\pi\pm x)=\mp\sin x,\quad \cos(\pi\pm x)=-\cos x,\quad \tan(\pi+x)=\tan x.$$

The graph of cos is the graph of sin translated by $\frac{\pi}{2}$ to the left: $\cos x=\sin\left(x+\frac{\pi}{2}\right)$.

Example 3

Show that $\cos\left(\dfrac{3\pi}{2}-x\right)=-\sin x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Expand	$\cos\frac{3\pi}{2}\cos x+\sin\frac{3\pi}{2}\sin x$
Step 2 Exact values	$0\times\cos x+(-1)\sin x$
Step 3 Simplify	$=-\sin x$

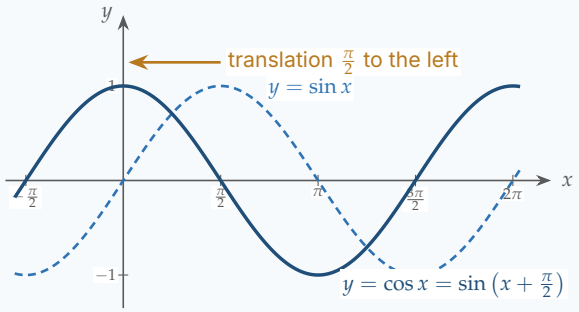


Figure 2 — $y=\sin\left(x+\frac{\pi}{2}\right)$ is $y=\sin x$ moved $\frac{\pi}{2}$ to the left, and it lies exactly on $y=\cos x$.

APPLY

1 $\sin\left(\frac{3\pi}{2}+x\right)$ in terms of $\cos x$.

2 $\tan(\pi-x)$ in terms of $\tan x$.

II.2 Proving an identity

Property 4 *proving an identity*

Start from the more complicated side, expand every compound angle, collect, and finish exactly on the other side. Pairs such as $\sin(x+c)+\sin(x-c)$ simplify: the $\cos x \sin c$ terms cancel, leaving $2\sin x \cos c$.

Example 4

Show that $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Start from the complicated side: double angles	$\dfrac{2\sin x \cos x}{1+(2\cos^2 x - 1)}$
Step 2 Simplify the denominator	$= \dfrac{2\sin x \cos x}{2\cos^2 x}$
Step 3 Finish on the other side	$= \dfrac{\sin x}{\cos x} = \tan x$

APPLY

3 Show that $\cos^4 x - \sin^4 x = \cos 2x$.

II.3 Equations with double angles

Example 5

Solve $\sin 2x = \sin x$, for $0 \le x \le 2\pi$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Double angle	$2\sin x \cos x - \sin x = 0$
Step 2 Factorise, do not divide	$\sin x\,(2\cos x - 1) = 0$
Step 3 Each factor	$\sin x = 0$: $0,\ \pi,\ 2\pi$; $\cos x = \frac{1}{2}$: $\frac{\pi}{3},\ \frac{5\pi}{3}$
Step 4 All the solutions	$0,\ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3},\ 2\pi$

Watch out 2

In an equation, keep every solution in the interval: dividing by $\sin x$ or $\cos x$ loses the solutions where it is 0. Factorise instead.

APPLY

4 Solve $\cos 2x + \cos x = 0$, for $0 \le x \le 2\pi$.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A The tangent of a sum

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 6]

The angles A and B are acute, with $\tan A = 2$ and $\tan B = 3$.

(a) Find the value of $\tan(A+B)$. [2]

(b) Hence find the exact value of $A+B$. [2]

(c) Find the value of $\tan 2A$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\tan(A+B)$ formula	$\dfrac{2+3}{1-2\times 3}$	M1
Step 2 Simplify	$\tan(A+B) = -1$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Place $A+B$	$0 < A+B < \pi$ and $\tan(A+B) < 0$, so $A+B$ is obtuse	R1
Step 2 The angle	$A+B = \dfrac{3\pi}{4}$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\tan 2A$ formula	$\dfrac{2\times 2}{1-2^2}$	M1
Step 2 Simplify	$\tan 2A = -\dfrac{4}{3}$	A1

B An identity, then an equation

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 7]

Consider the expression $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)$, for $x \in \mathbb{R}$.

(a) Show that $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=\sin x$. [3]

(b) Hence solve the equation $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=\cos 2x$, for $0 \le x \le 2\pi$. [4]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Expand both	$\sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3} + \sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3}$ *Both expansions.*	M1
Step 2 The cross terms cancel	$=2\sin x\cos\frac{\pi}{3}$	A1
Step 3 $\cos\frac{\pi}{3}=\frac{1}{2}$	$=2\times\frac{1}{2}\sin x=\sin x$ *AG.*	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\cos 2x=1-2\sin^2 x$	$\sin x=1-2\sin^2 x$ *Using* $\cos 2x=1-2\sin^2 x$.	M1
Step 2 Factorise	$(2\sin x-1)(\sin x+1)=0$ *Or* $\sin x=\frac{1}{2}$, $\sin x=-1$.	A1
Step 3 $\sin x=\frac{1}{2}$	$x=\frac{\pi}{6},\ x=\frac{5\pi}{6}$	A1
Step 4 $\sin x=-1$	$x=\frac{3\pi}{2}$	A1

C On the GDC (Paper 2)

EXAM-STYLE QUESTION 3 Paper 2 · calculator · [Maximum mark: 6]

Let $f(x) = 2\sin\left(x + \frac{\pi}{6}\right) + \cos 2x$, for $0 \leq x \leq 2\pi$.

(a) Show that $f(x) = \sqrt{3}\sin x + \cos x + \cos 2x$. [2]

(b) Find the maximum value of f, and the value of x at which it occurs. [2]

(c) Solve the equation $f(x) = 0$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Expand	$2\left(\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}\right) + \cos 2x$	M1
Step 2 Exact values	$= 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) + \cos 2x = \sqrt{3}\sin x + \cos x + \cos 2x$ *AG.*	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Maximum on the GDC	2.30 $(2.29813\ldots)$	A1
Step 2 Where	at $x = 0.349$ $(\frac{\pi}{9})$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 First zero	$x = \pi$ (3.14)	A1
Step 2 Second zero	$x = 5.63$	A1

IV WHICH METHOD? READ THE QUESTION

3.11 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
$\sin(A \pm B)$, $\cos(A \pm B)$	the compound identities	expand; the sign changes only in $\cos$ P1
$\tan A$ *and* $\tan B$ *given*	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	substitute, then simplify P1
"hence find $A + B$*"*	where $A + B$ lies	the angle in that interval with that tangent W1
$\tan 2A$	$\frac{2\tan A}{1-\tan^2 A}$	one ratio is enough P2
the exact value of $\sin \frac{\pi}{12}$, $\cos \frac{5\pi}{12}$ …	a sum or difference of known angles	$\frac{\pi}{3} - \frac{\pi}{4}$, $\frac{\pi}{4} + \frac{\pi}{6}$, then expand P2
$\sin\left(\frac{\pi}{2} - x\right)$, $\cos(\pi + x)$…	the symmetries	$\cos x$, $-\cos x$…, or expand to check P3
"show that" with $x \pm c$	expand both sides	the cross terms cancel P4
"hence solve" with $\cos 2x$	a quadratic in one ratio	$1 - 2\sin^2 x$ or $2\cos^2 x - 1$, factorise W2

In the exam 1 *command terms*

Show that: start from one side and end exactly on the other, every step written. **Hence**: use the identity just shown. **Find the exact value**: no decimal; an angle as a multiple of π.

V ON THE GDC: PAPER 2

AHL 3.10 · 3.11 · NumWorks · every calculator move of this topic

1 Check an identity on the graph

Grapher → f(x)=sin(x+π/3)+sin(x-π/3) → g(x)=sin(x)

Before a *show that*, draw both sides: if the curves coincide on the whole window, the identity is worth proving. A graph is not a proof.

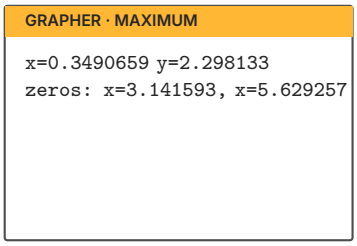
GRAPHER

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y1=sin(x+π/3)+sin(x-π/3)
y2=sin(x)
the two curves coincide
```

2 A maximum, then the zeros

Grapher → f(x)=2sin(x+π/6)+cos(2x) → x: 0 to 2π → Maximum → Zeros

The window covers $0 \le x \le 2\pi$; read the maximum, then every zero. **On paper:** each value to three significant figures, and $x = \pi$ exact when you see it.



ANSWERS TO THE APPLY QUESTIONS

I.1 $\frac{\sqrt{6}+\sqrt{2}}{4}$

I.2 $\frac{48}{65} + \frac{15}{65} = \frac{63}{65}$

I.3 $\frac{24}{25}$; $\frac{7}{25}$

I.4 $\dfrac{1+t}{1-t}$

II.1 $-\cos x$

II.2 $-\tan x$

II.3 $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos 2x \times 1$

II.4 $(2\cos x - 1)(\cos x + 1) = 0$: $\frac{\pi}{3}$, π, $\frac{5\pi}{3}$

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