Compound angle identities

Skills sheet · HL AA · AHL 3.10 · 3.11

KEY FACTS

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ P1

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ P1

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$, $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ P2

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$, $\cos(\pi + x) = -\cos x$ P3

$\sin(A + B) \neq \sin A + \sin B$ P1.

Place $A + B$ before the angle W1.

Factorise, never divide by $\sin x$ W2.

WHAT UNLOCKS

- $\sin\frac{\pi}{12}$, $\cos\frac{5\pi}{12}$, $\tan\frac{\pi}{12}$ $\rightarrow$ a sum or difference of $\frac{\pi}{3}$, $\frac{\pi}{4}$, $\frac{\pi}{6}$.
- $\sin A$ and $\cos B$ given $\rightarrow$ the other two ratios from $\cos^2 + \sin^2 = 1$, then expand.
- $\sin 3x \cos x - \cos 3x \sin x$ $\rightarrow$ read the identity backwards: $\sin 2x$.
- "show that" with $x \pm c$ $\rightarrow$ expand both, the cross terms cancel.

WORKED EXAMPLE ①

Find the exact value of $\sin\frac{\pi}{12}$. no GDC

$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$ M1

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ A1

Method

An exact compound value

① Split into known angles.
② Expand with the identity.
③ Multiply the surds, one fraction.

WORKED EXAMPLE ②

Show that $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$. no GDC

$\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x$ M1

$= 0 \times \cos x - 1 \times \sin x = -\sin x$ A1

Method

A symmetry

① Expand the compound angle.
② Exact values of $\frac{\pi}{2}$ or π.
③ End exactly on the other side.

FLUENCY

1. Exact value of $\cos\frac{5\pi}{12}$ no GDC

2. Exact value of $\tan\frac{\pi}{12}$ no GDC

3. Simplify $\sin(\pi - x)$ no GDC

4. Simplify $\cos(\pi + x)$ no GDC

5. Simplify $\tan\left(\frac{\pi}{2} - x\right)$ no GDC

6. Simplify $\sin 3x \cos x - \cos 3x \sin x$ no GDC

REASONING

●●● **Question 1.** The angles A and B are acute, with $\sin A = \frac{1}{3}$ and $\cos B = \frac{2}{3}$. [no GDC]

(a) Find the exact value of $\sin(A + B)$. [4]

●●● **Question 2.** It is given that $\tan(A + B) = \frac{1}{2}$ and $\tan A = 3$. [no GDC]

(a) Find the value of $\tan B$. [3]

●●● **Question 3.** [no GDC]

(a) Show that $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$. [3]

PROBLEMS

●●● **Question 4.** [no GDC]

(a) Solve the equation $\sin\left(x + \frac{\pi}{6}\right) = 2\cos x$, for $0 \leq x \leq 2\pi$. [4]

●●● **Question 5.** [no GDC]

(a) Solve the equation $\tan 2\theta = 3\tan\theta$, for $0 < \theta < \pi$. [4]

ANSWERS

1 $\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$

2 $\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\sqrt{3}-1}{1+\sqrt{3}}=2-\sqrt{3}$

3 $\sin x$

4 $-\cos x$

5 $\cot x$

6 $\sin 2x$

Q1 $\cos A=\frac{2\sqrt{2}}{3}$, $\sin B=\frac{\sqrt{5}}{3}$; $\sin(A+B)=\frac{1}{3}\cdot\frac{2}{3}+\frac{2\sqrt{2}}{3}\cdot\frac{\sqrt{5}}{3}=\frac{2+2\sqrt{10}}{9}$

Q2 $\frac{3+\tan B}{1-3\tan B}=\frac{1}{2}$: $6+2\tan B=1-3\tan B$, $\tan B=-1$

Q3 $\sin^2 x\cos^2 y-\cos^2 x\sin^2 y=\sin^2 x\left(1-\sin^2 y\right)-\left(1-\sin^2 x\right)\sin^2 y=\sin^2 x-\sin^2 y$

Q4 $\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=2\cos x$: $\tan x=\sqrt{3}$ ($\cos x=0$ is not a solution): $x=\frac{\pi}{3}$, $x=\frac{4\pi}{3}$

Q5 $\frac{2t}{1-t^2}=3t$ with $t=\tan\theta$: $t=0$ (no θ in $]0,\pi[$) or $t^2=\frac{1}{3}$: $\theta=\frac{\pi}{6}$, $\theta=\frac{5\pi}{6}$

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