Inverse trigonometric functions

Skills sheet · HL AA · AHL 3.9

KEY FACTS

Ranges: arcsin, arctan in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, arccos in $[0, \pi]$ D1

$\arcsin(\sin x) = x$ only inside the range P1

$y = \arctan x$: asymptotes $y = \pm\frac{\pi}{2}$ P2

Domain: $-1 \le ax + b \le 1$; $f^{-1} = \frac{1}{b}\arcsin\frac{x-d}{a}$ P3

The answer must be in the range W1.

$\sin^{-1} x \neq \frac{1}{\sin x}$ W1.

Divide before arcsin W2.

WHAT UNLOCKS

- $\arcsin(\sin a)$, $\arccos(\cos a)$ → the angle with the same ratio, inside the range.
- $\cos(\arcsin x)$ → $\alpha = \arcsin x$, then $\cos\alpha = \sqrt{1-x^2}$, positive.
- "the domain of $\arccos(ax+b)$" → $-1 \le ax + b \le 1$, solved.
- $f(x) = a\sin(bx) + d$ restricted → $f^{-1}(x) = \frac{1}{b}\arcsin\frac{x-d}{a}$, domain the range of f.

WORKED EXAMPLE ①

Find the exact value of $\arcsin\left(\sin\frac{7\pi}{6}\right)$. no GDC

$\sin\frac{7\pi}{6} = -\frac{1}{2}$ M1

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ A1

Method

An inverse of a ratio

① Work out the inside first.
② Find the angle in the range.
③ Check: it is not $\frac{7\pi}{6}$.

WORKED EXAMPLE ②

Find the domain and range of $f(x) = \arccos(3x - 2)$. no GDC

$-1 \le 3x - 2 \le 1$ M1

$\frac{1}{3} \le x \le 1$; range $0 \le f(x) \le \pi$ A1A1

Method

A domain

① The inside between -1 and 1.
② Solve the double inequality.
③ Range: that of arccos.

FLUENCY

●○○ **1.** $\arcsin\frac{\sqrt{3}}{2}$ no GDC

●○○ **2.** $\arccos 0$ no GDC

●○○ **3.** $\arctan\frac{1}{\sqrt{3}}$ no GDC

●○○ **4.** $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ no GDC

●○○ **5.** $\arccos(-1)$ no GDC

●○○ **6.** $\arctan(-1)$ no GDC

REASONING

●●● **Question 1.** [no GDC]

(a) Find the exact value of $\cos\left(\arcsin\frac{3}{7}\right)$. [3]

●●● **Question 2.** [no GDC]

(a) Find the exact value of $\arccos\left(\cos\frac{4\pi}{3}\right)$. [2]

●●● **Question 3.** Let $f(x) = 2\arctan(x-1)$, for $x \in \mathbb{R}$. [no GDC]

(a) State the range of f. [1]

(b) Solve the equation $f(x) = \frac{\pi}{2}$. [2]

PROBLEMS

●●● **Question 4.** Let $f(x) = \frac{\pi}{2} + \arcsin(2x-1)$. [no GDC]

(a) Find the largest possible domain of f, and the range of f on it. [3]

(b) Solve the equation $f(x) = \frac{5\pi}{6}$. [2]

●●● **Question 5.** Let $f(x) = 4\tan\left(\frac{x}{2}\right) - 3$, for $-\pi < x < \pi$. [no GDC]

(a) Find $f^{-1}(x)$, and state its domain and range. [3]

ANSWERS

1 $\frac{\pi}{3}$

2 $\frac{\pi}{2}$

3 $\frac{\pi}{6}$

4 $-\frac{\pi}{4}$

5 π

6 $-\frac{\pi}{4}$

Q1 $\alpha = \arcsin\frac{3}{7}$: $\cos\alpha = \sqrt{1-\frac{9}{49}} = \frac{\sqrt{40}}{7} = \frac{2\sqrt{10}}{7}$, positive since α is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Q2 $\cos\frac{4\pi}{3} = -\frac{1}{2} = \cos\frac{2\pi}{3}$ and $\frac{2\pi}{3} \in [0, \pi]$: $\frac{2\pi}{3}$

Q3 $-\pi < f(x) < \pi$; $\arctan(x-1) = \frac{\pi}{4}$: $x - 1 = 1$, $x = 2$

Q4 $-1 \le 2x - 1 \le 1$: $0 \le x \le 1$, range $0 \le f(x) \le \pi$; $\arcsin(2x-1) = \frac{\pi}{3}$: $x = \frac{2+\sqrt{3}}{4}$

Q5 $\tan\frac{y}{2} = \frac{x+3}{4}$: $f^{-1}(x) = 2\arctan\frac{x+3}{4}$, domain $\mathbb{R}$, range $-\pi < f^{-1}(x) < \pi$

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