Reciprocal trigonometric functions

Skills sheet · HL AA · AHL 3.9

KEY FACTS

$\sec x = \frac{1}{\cos x}$, $\operatorname{cosec} x = \frac{1}{\sin x}$, $\cot x = \frac{\cos x}{\sin x}$ D1

Each has the sign of its partner P1

$1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \operatorname{cosec}^2 x$ P2

$\tan^2 x$ with $\sec x$: a quadratic in $\sec x$ P3

$\sec x \neq \cos^{-1} x$ W1.

The quadrant chooses the sign W2.

$|\sec x| \geq 1$, $|\operatorname{cosec} x| \geq 1$ P1.

WHAT UNLOCKS

- "exact value of sec, cosec, cot" $\rightarrow$ the exact cos, sin, tan, then one over it.
- $\tan\theta$ given, $\sec\theta$ wanted $\rightarrow$ $1 + \tan^2\theta$, a square root, the sign from the quadrant.
- $\sec x - \cos x$, $\frac{1+\sec x}{\ldots}$ $\rightarrow$ write everything in sin and cos, one fraction.
- an equation with $\cot^2 x$ and $\operatorname{cosec} x$ $\rightarrow$ replace $\cot^2 x$ by $\operatorname{cosec}^2 x - 1$.

WORKED EXAMPLE ①

Find the exact value of $\operatorname{cosec} \frac{5\pi}{4}$**.** no GDC

$\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ M1

$\operatorname{cosec} \frac{5\pi}{4} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$ A1

Method

An exact reciprocal

① Reference angle and sign.
② The exact sin, cos or tan.
③ One over it; simplify the surd.

WORKED EXAMPLE ②

Solve $\tan^2 x + \sec x = 1$**, for** $0 \leq x < 2\pi$**.** no GDC

$\sec^2 x - 1 + \sec x = 1$, $(\sec x + 2)(\sec x - 1) = 0$ M1A1

$\cos x = -\frac{1}{2}$ or $\cos x = 1$: $x = 0$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$ A1

Method

A quadratic in sec

① $\tan^2 x = \sec^2 x - 1$.
② Factorise in $\sec x$.
③ Back to $\cos x$; every x in the interval.

FLUENCY

●○○ **1.** $\sec \frac{\pi}{4}$ no GDC

●○○ **2.** $\operatorname{cosec} \frac{\pi}{3}$ no GDC

●○○ **3.** $\cot \frac{3\pi}{4}$ no GDC

●○○ **4.** $\sec \pi$ no GDC

●○○ **5.** $\operatorname{cosec} \frac{3\pi}{2}$ no GDC

●○○ **6.** $\cot \frac{\pi}{6}$ no GDC

REASONING

●●● **Question 1.** It is given that $\sin\theta = -\frac{2}{3}$, where $\pi < \theta < \frac{3\pi}{2}$. [no GDC]

(a) Find the exact values of $\operatorname{cosec}\theta$ and $\cot\theta$. [3]

●●● **Question 2.** [no GDC]

(a) Show that $\sec x - \cos x = \sin x \tan x$. [2]

●●● **Question 3.** It is given that $\cot\theta = 3$, where $\pi < \theta < \frac{3\pi}{2}$. [no GDC]

(a) Find the exact value of $\operatorname{cosec}\theta$. [3]

PROBLEMS

●●● **Question 4.** Consider the equation $2\sec^2 x - 3\tan x - 3 = 0$, for $0 \leq x \leq \pi$, $x \neq \frac{\pi}{2}$. [with GDC]

(a) Write the equation as a quadratic in $\tan x$. [2]

(b) Hence solve the equation. [3]

●●● **Question 5.** [no GDC]

(a) Prove that $\dfrac{1+\sec x}{\tan x + \sin x} = \operatorname{cosec} x$, for every x where both sides are defined. [3]

ANSWERS

1 $\sqrt{2}$

2 $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

3 -1

4 -1

5 -1

6 $\sqrt{3}$

Q1 $\operatorname{cosec}\theta = -\frac{3}{2}$; $\cos\theta = -\frac{\sqrt{5}}{3}$ (third quadrant), so $\cot\theta = \frac{\sqrt{5}}{2}$

Q2 $\frac{1}{\cos x} - \cos x = \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \sin x \tan x$

Q3 $\operatorname{cosec}^2\theta = 1 + 9 = 10$; $\sin\theta < 0$ in the third quadrant: $\operatorname{cosec}\theta = -\sqrt{10}$

Q4 $2(1 + \tan^2 x) - 3\tan x - 3 = 0$: $2\tan^2 x - 3\tan x - 1 = 0$; $\tan x = \frac{3 \pm \sqrt{17}}{4} = 1.78$ or -0.281: $x = 1.06$, $x = 2.87$

Q5 $\frac{1 + \frac{1}{\cos x}}{\frac{\sin x}{\cos x} + \sin x} = \frac{\cos x + 1}{\sin x(1 + \cos x)} = \frac{1}{\sin x} = \operatorname{cosec} x$

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