Radians, arcs and sectors

Degrees cut a turn into 360 for convenience; radians measure an angle by the arc it cuts on a circle of radius 1. With that choice the formulas lose their awkward $\frac{\pi}{180}$, and calculus with sin and cos becomes possible.

I RADIAN MEASURE

SL 3.4 · the radian, converting degrees and radians

I.1 The radian

Definition 1 *radian*

An angle of **one radian** at the centre of a circle cuts an arc as long as the radius. A full turn is 2π radians: π radians $= 180°$.

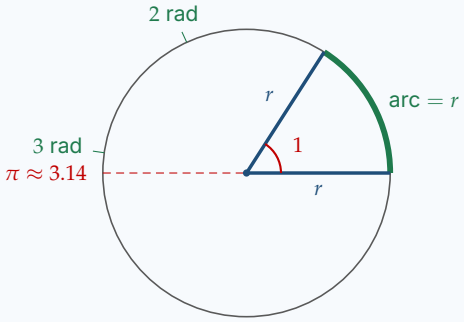


Figure 1 — Arcs of length r laid around a circle: π of them reach half way round, so one radian is about 57.3° — Example 1.

Watch out 1

An angle with no degree sign is in radians: $\sin 2$ is the sine of 2 radians. In Paper 2, set the calculator to radians.

Example 1

Show that one radian is about 57.3°.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Start from π rad $= 180°$	π rad $= 180°$
Step 2 Divide by π	$1 \text{ rad} = \dfrac{180°}{\pi}$
Step 3 Compute	$57.2957\ldots° \approx 57.3°$

APPLY

1 2 radians in degrees.

2 0.5 radians in degrees.

I.2 Degrees to radians

Property 1 *converting*

Degrees to radians: $\times \dfrac{\pi}{180}$. Radians to degrees: $\times \dfrac{180}{\pi}$. By heart: $30° = \frac{\pi}{6}$, $45° = \frac{\pi}{4}$, $60° = \frac{\pi}{3}$, $90° = \frac{\pi}{2}$.

Example 2

Convert $210°$ to radians, in terms of π.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Multiply by $\dfrac{\pi}{180}$	$210 \times \dfrac{\pi}{180}$
Step 2 Simplify the fraction	$\dfrac{210}{180} = \dfrac{7}{6}$: $\dfrac{7\pi}{6}$

APPLY

3 $40°$

4 $300°$

5 $18°$

I.3 Radians to degrees

Example 3

Convert $\dfrac{5\pi}{12}$ radians and 2.4 radians to degrees.

WHAT YOU DO	WHAT YOU WRITE
Step 1 With π: replace π by $180°$	$\dfrac{5 \times 180°}{12} = 75°$
Step 2 Without π: multiply by $\dfrac{180}{\pi}$	$2.4 \times \dfrac{180}{\pi} = 137.5°$

APPLY

6 $\frac{4\pi}{3}$

7 $\frac{\pi}{8}$

8 3 rad

II ARCS, SECTORS AND SEGMENTS

SL 3.4 · arc length, area of a sector, segments

II.1 Arc length and perimeter

Property 2 *arc length and sector area*

Sector of radius r and angle θ **in radians**: $l = r\theta$, $A = \frac{1}{2}r^2\theta$. The perimeter of the sector is $2r + r\theta$.

Example 4

A sector has radius 7 cm and angle 0.9 radians. Find the arc length and the perimeter.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Arc: $l = r\theta$	$7 \times 0.9 = 6.3$ cm
Step 2 Perimeter: two radii and the arc	$7 + 7 + 6.3 = 20.3$ cm

APPLY

1 $r = 12$ cm, $\theta = \frac{\pi}{4}$: the arc.

2 $r = 4.5$ cm, $\theta = 1.6$: the perimeter.

3 Arc 15.6 cm, $r = 6$ cm: the angle.

II.2 The area of a sector

Watch out 2

Both formulas need θ in radians: with degrees, convert first. The perimeter of a sector includes the two radii.

Example 5

A sector has radius 6 cm and angle 50°. Find its area.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Convert to radians	$50 \times \dfrac{\pi}{180} = \dfrac{5\pi}{18}$
Step 2 $A = \frac{1}{2}r^2\theta$	$\frac{1}{2} \times 36 \times \dfrac{5\pi}{18} = 5\pi$
Step 3 Round at the end	15.7 cm^2

APPLY

4 $r = 10$ cm, $\theta = 0.75$: the area.

5 $r = 5$ cm, $\theta = 1.4$: the area.

6 Area 24 cm^2, $r = 4$ cm: the angle.

II.3 The area of a segment

Definition 2 *segment*

A **segment** lies between a chord and its arc: sector minus triangle, $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{1}{2}r^2(\theta - \sin\theta)$.

Example 6

A chord cuts off an angle of 1.2 radians at the centre of a circle of radius 5 cm. Find the area of the minor segment.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Sector	$\frac{1}{2} \times 25 \times 1.2 = 15$
Step 2 Triangle: $\frac{1}{2}r^2\sin\theta$, in radians	$\frac{1}{2} \times 25 \times \sin 1.2 = 11.65\ldots$
Step 3 Subtract	$3.35\ \text{cm}^2$

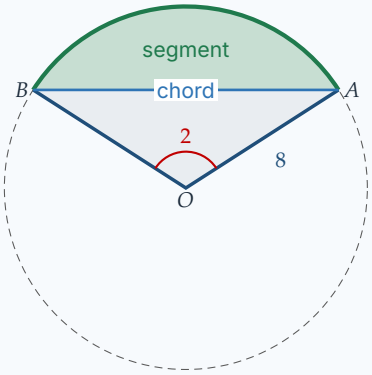


Figure 2 — Exam-style question 3: radius 8, angle 2 radians — sector 64, triangle 29.1, segment 34.9, chord 13.5.

APPLY

7 $r = 10$ cm, $\theta = \frac{\pi}{2}$: the segment.

8 $r = 3$ cm, $\theta = 2.5$: the segment.

II.4 Finding θ or r

Example 7

A sector of radius 8 cm has perimeter 26 cm. Find its angle and its area.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Arc: perimeter minus the two radii	$l = 26 - 16 = 10$
Step 2 $\theta = \dfrac{l}{r}$	$\theta = \dfrac{10}{8} = 1.25$
Step 3 Area	$\frac{1}{2} \times 64 \times 1.25 = 40\ \text{cm}^2$

APPLY

9 A sector has area 20 cm^2 and arc 8 cm. Find r and θ.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Converting angles

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 4]

This question is about converting angles between degrees and radians.

(a) Convert $150°$ to radians, giving your answer in terms of π. [2]

(b) Convert $\frac{7\pi}{4}$ radians to degrees. [1]

(c) Convert $\frac{\pi}{12}$ radians to degrees. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Multiply by $\frac{\pi}{180}$	$150 \times \frac{\pi}{180}$	(M1)
Step 2 Simplify the fraction	$\frac{5\pi}{6}$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Replace π by $180°$	$315°$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Replace π by $180°$	$15°$	A1

B Arcs and sectors

EXAM-STYLE QUESTION 2 — Paper 1 · no calculator · [Maximum mark: 5]

A sector of a circle has radius 10 cm and angle 1.2 radians.

(a) Find the length of the arc. [2]

(b) Find the perimeter of the sector. [1]

(c) Find the area of the sector. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $l = r\theta$	$l = 10 \times 1.2$	(M1)
Step 2 Compute	12 cm	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Two radii and the arc	$10 + 10 + 12 = 32$ cm *The arc and the two radii.*	A1

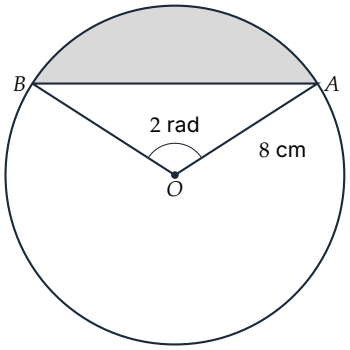
WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $A = \frac{1}{2}r^2\theta$	$\frac{1}{2} \times 10^2 \times 1.2$	(M1)
Step 2 Compute	60 cm^2	A1

C Segments (Paper 2)

EXAM-STYLE QUESTION 3 Paper 2 · calculator · [Maximum mark: 6]

The diagram shows a circle with centre O and radius 8 cm. The chord AB subtends an angle of 2 radians at O. The shaded region is the minor segment.



(a) Find the area of the sector OAB. [1]
(b) Find the area of the triangle OAB. [2]
(c) Find the area of the shaded segment. [1]
(d) Find the length of the chord AB. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Sector: $\frac{1}{2}r^2\theta$	$\frac{1}{2} \times 8^2 \times 2 = 64 \text{ cm}^2$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Triangle: $\frac{1}{2}r^2 \sin\theta$, radians	$\frac{1}{2} \times 8^2 \times \sin 2$ *The calculator in radians.*	(M1)
Step 2 Compute	$29.1 \text{ cm}^2 \quad (29.0975\ldots)$	A1

WORKING (c)

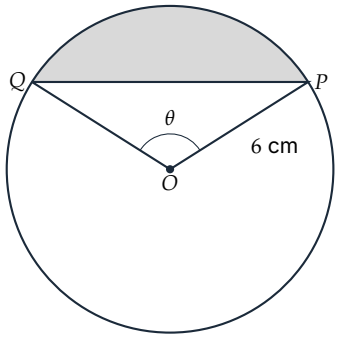
WHAT YOU DO	WHAT YOU WRITE	
Step 1 Sector minus triangle	$64 - 29.0975\ldots = 34.9 \text{ cm}^2$	A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Cosine rule in triangle OAB	$AB^2 = 8^2 + 8^2 - 2 \times 8 \times 8\cos 2$ *Or $AB = 2 \times 8 \sin 1$.*	(M1)
Step 2 Compute	$AB = 13.5 \text{ cm} \quad (13.4635\ldots)$	A1

EXAM-STYLE QUESTION 4 Paper 2 · calculator · [Maximum mark: 6]

A circle has centre O and radius 6 cm. The chord PQ subtends an angle of θ radians at O, where $0 < \theta < \pi$. The area of the minor segment cut off by PQ is 20 cm^2.



(a) Show that $\theta - \sin\theta = \dfrac{10}{9}$. [2]

(b) Find the value of θ. [2]

(c) Find the length of the arc PQ. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Segment = sector minus triangle	$\frac{1}{2} \times 6^2\theta - \frac{1}{2} \times 6^2 \sin\theta = 20$	M1
	Segment = sector − triangle.	
Step 2 Equate to 20	$18(\theta - \sin\theta) = 20$	A1
Step 3 Divide by 18	$\theta - \sin\theta = \dfrac{10}{9}$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Graph both sides on the GDC	a graph of $y = \theta - \sin\theta$ and $y = \frac{10}{9}$, or a solver	(M1)
Step 2 Intersection	$\theta = 2.01$ $(2.01434\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $l = r\theta$	$6 \times 2.01434\ldots$	(M1)
Step 2 Compute	12.1 cm $(12.0860\ldots)$	A1

IV WHICH METHOD? READ THE QUESTION

3.4 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
an angle in degrees, radians wanted	$\times \frac{\pi}{180}$	simplify the fraction of π P1
an angle in radians, degrees wanted	$\times \frac{180}{\pi}$	π becomes $180°$ P1
"length of the arc"	$l = r\theta$	θ in radians P2
"perimeter of the sector"	two radii and the arc	$2r + r\theta$ P2
"area of the sector"	$A = \frac{1}{2}r^2\theta$	θ in radians P2
"shaded segment"	sector minus triangle	$\frac{1}{2}r^2(\theta - \sin\theta)$ D2
area or arc given, θ wanted	an equation in θ	solve; with $\sin\theta$ in it, the GDC

In the exam 1 *command terms*

In terms of π: an exact answer such as $\frac{5\pi}{6}$. **Find**: the formula with its numbers, then the answer to 3 s.f. with units. **Show that**: the equation set up, then rearranged to the given form.

V ON THE GDC: PAPER 2

SL 3.4 · NumWorks · every calculator move of this topic

1 Radians first

Settings → Angle measure → Radians

Sectors and segments use $\sin\theta$ with θ in radians. The banner must show rad: $\sin 2$ is 0.909 in radians, 0.0349 in degrees.

SETTINGS

```
Angle measure
Degrees
> Radians
```

2 A triangle with $\sin\theta$

Calculation → 0.5×8^2×sin(2) → EXE

On paper: "$\frac{1}{2} \times 8^2 \sin 2 = 29.1$".

CALCULATION

```
0.5*8^2*sin(2)
29.09751766
```

3 An angle from an equation

Grapher → f(x)=x−sin(x) → g(x)=10/9 → Find → Intersection

An equation with both θ and $\sin\theta$ cannot be solved by algebra: graph both sides and find where they meet. **On paper:** the equation, then $\theta = 2.01$.

GRAPHER · INTERSECTION

```
f(x)=x-sin(x)
g(x)=10/9
x=2.014345 y=1.111111
```

ANSWERS TO THE APPLY QUESTIONS

I.1 $114.6°$

I.2 $28.6°$

I.3 $\frac{2\pi}{9}$

I.4 $\frac{5\pi}{3}$

I.5 $\frac{\pi}{10}$

I.6 $240°$

I.7 $22.5°$

I.8 $171.9°$

II.1 3π cm

II.2 16.2 cm

II.3 2.6 rad

II.4 37.5 cm^2

II.5 17.5 cm^2

II.6 3 rad

II.7 $25\pi - 50 = 28.5$ cm^2

II.8 8.56 cm^2

II.9 $A = \frac{1}{2}rl$: $r = 5, \theta = 1.6$

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