Solids and 3D geometry

Skills sheet · SL AA 3.1

KEY FACTS

$AB = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$; midpoint: the means D1
Cone $\frac{1}{3}\pi r^2 h$, curved $\pi r l$; sphere $\frac{4}{3}\pi r^3$, $4\pi r^2$ P1
Pyramid $\frac{1}{3}Ah$; hemisphere $\frac{2}{3}\pi r^3$, curved $2\pi r^2$ P1
Line and plane: the projection, then $\tan\theta$ D2 P2

V uses h; the curved surface uses l W1.
Combined solids: outside faces only.
The angle with the projection, not with an edge W2.

WHAT UNLOCKS

- two points → Pythagoras in three directions.
- "centre of a sphere on a diameter" → the midpoint, radius half of AB.
- a cone with r and h → $l = \sqrt{r^2 + h^2}$ before the surface.
- "angle with the base" → the foot of the perpendicular, a right-angled triangle.

WORKED EXAMPLE ①

Find the distance between $(1, 2, 2)$ **and** $(3, -1, 8)$. no GDC

$\sqrt{2^2 + (-3)^2 + 6^2}$ M1
$= \sqrt{49} = 7$ A1

Method
A distance in 3D
① The three differences.
② Square and add.
③ Square root.
$(-3)^2 = 9$, ***not*** -9.

WORKED EXAMPLE ②

Find the exact volume of a hemisphere of radius 3. no GDC

$\frac{2}{3}\pi \times 3^3$ M1
$= 18\pi$ A1

Method
A volume
① Find the formula (booklet).
② Substitute.
③ Exact: keep π.

FLUENCY

●○○ **1.** Midpoint of $(2, 4, -6)$ and $(0, -2, 2)$ no GDC

●○○ **2.** Distance from $(0, 0, 0)$ to $(2, 3, 6)$ no GDC

●○○ **3.** Volume of a cone, $r = 3$, $h = 4$, in terms of π no GDC

●○○ **4.** Slant height of the same cone no GDC

●○○ **5.** Surface area of a sphere, $r = 2$ no GDC

●○○ **6.** Volume of a pyramid, square base 6, height 5 no GDC

REASONING

●●● **Question 1.** A sphere has volume 500 cm^3. [with GDC]

(a) Find its radius. [2]

(b) Find its surface area. [2]

●●● **Question 2.** A cube has edges of length 4. [with GDC]

(a) Find the angle between a diagonal of the cube and its base. [3]

●●● **Question 3.** A cone has radius 6 and slant height 10. [no GDC]

(a) Find its height and its exact volume. [3]

PROBLEMS

●●● **Question 4.** A solid hemisphere of radius 6 has the same volume as a cone of radius 6. [no GDC]

(a) Find the height of the cone. [3]

●●● **Question 5.** A right pyramid has a square base of side 8 and height 6. M is the midpoint of a side of the base. [with GDC]

(a) Find the angle between VM and the base, where V is the apex. [3]

ANSWERS

1 $(1,\ 1,\ -2)$

2 7

3 12π

4 5

5 16π

6 60

Q1 (a) 4.92 cm (b) 305 cm^2

Q2 $\tan\theta = \frac{4}{4\sqrt{2}},\ 35.3^\circ$

Q3 $h = 8,\ V = 96\pi$

Q4 $\frac{2}{3}\pi \times 6^3 = \frac{1}{3}\pi \times 6^2 h,\ h = 12$

Q5 $\tan\theta = \frac{6}{4},\ 56.3^\circ$

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