The unit circle and identities

Skills sheet · SL AA 3.5 · 3.6

KEY FACTS

$P(\cos\theta, \sin\theta)$; $\tan\theta = \frac{\sin\theta}{\cos\theta}$ D1

Exact values of $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$; symmetries P1

Ambiguous case: C or $180° - C$ P2

$\cos^2\theta + \sin^2\theta = 1$; $\sin 2\theta$, $\cos 2\theta$ P3 P4

The quadrant chooses the sign W2.

arcsin *gives only the acute angle* W1.

$\sin 2\theta \neq 2\sin\theta$.

WHAT UNLOCKS

- an exact value → the reference angle, then the sign of the quadrant.
- one ratio → the other two by $\cos^2 + \sin^2 = 1$.
- $\sin 2\theta$, $\cos 2\theta$ → the double angle identities.
- two sides and a non-included angle → check for two triangles.

WORKED EXAMPLE ①

Find the exact value of $\cos\frac{4\pi}{3}$**.** no GDC

reference angle $\frac{\pi}{3}$, third quadrant — M1

$-\frac{1}{2}$ — A1

Method

An exact value

① The reference angle.
② Its exact value.
③ The sign from the quadrant.

WORKED EXAMPLE ②

$\sin\theta = 0.6$**,** θ **acute. Find** $\cos\theta$ **and** $\tan\theta$**.** no GDC

$\cos\theta = \sqrt{1 - 0.36} = 0.8$ — M1A1

$\tan\theta = \frac{0.6}{0.8} = 0.75$ — A1

Method

Another ratio

① $\cos^2\theta = 1 - \sin^2\theta$.
② The sign: θ acute.
③ $\tan = \frac{\sin}{\cos}$.

Two signs for the root: choose.

FLUENCY

1. $\sin\frac{\pi}{3}$ no GDC

2. $\cos\frac{3\pi}{4}$ no GDC

3. $\tan\frac{\pi}{6}$ no GDC

4. $\sin\left(-\frac{\pi}{6}\right)$ no GDC

5. $\cos\pi$ no GDC

6. The sign of $\tan\theta$ for $\pi < \theta < \frac{3\pi}{2}$ no GDC

REASONING

●●● **Question 1.** $\cos\theta = \frac{5}{13}$ and $\frac{3\pi}{2} < \theta < 2\pi$. [no GDC]

(a) Find $\sin\theta$ and $\tan\theta$. [3]

●●● **Question 2.** Show that $\dfrac{1-\cos 2\theta}{\sin 2\theta} = \tan\theta$. [no GDC]

(a) Prove the identity. [3]

●●● **Question 3.** $\sin\theta = 0.6$ with θ acute. [no GDC]

(a) Find $\sin 2\theta$ and $\cos 2\theta$. [3]

PROBLEMS

●●● **Question 4.** In a triangle ABC, $AC = 8$, $BC = 6$ and $B\hat{A}C = 35°$. [with GDC]

(a) Find the two possible values of $A\hat{B}C$. [4]

●●● **Question 5.** $\tan\theta = 2$ with θ acute. [no GDC]

(a) Find the exact value of $\sin 2\theta$. [3]

ANSWERS

1 $\frac{\sqrt{3}}{2}$

2 $-\frac{\sqrt{2}}{2}$

3 $\frac{1}{\sqrt{3}}$

4 $-\frac{1}{2}$

5 -1

6 positive

Q1 $\sin\theta = -\frac{12}{13},\ \tan\theta = -\frac{12}{5}$

Q2 $\frac{2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta}$

Q3 0.96 and 0.28

Q4 $\sin B = \frac{8\sin 35^\circ}{6}$: 49.9° or 130.1°

Q5 $\sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$: $\frac{4}{5}$

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