Triangle trigonometry

Any triangle can be solved from three of its measurements, as long as one is a side. The choice of rule depends only on what you are given: a side with its opposite angle calls for the sine rule, an angle between two known sides for the cosine rule.

I SOLVING A TRIANGLE

SL 3.2 · right-angled ratios, sine rule, cosine rule, area

I.1 A right-angled triangle

Definition 1 *the ratios in a right-angled triangle*

For an acute angle θ of a right-angled triangle: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.

Example 1

In the triangle ABC, $A\hat{C}B = 90°$, $AB = 15$ cm and $B\hat{A}C = 40°$. Find BC and AC.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Label the sides from the angle A	BC opposite, AC adjacent, $AB = 15$ hypotenuse
Step 2 Opposite and hypotenuse: sin	$BC = 15\sin 40° = 9.64$ cm
Step 3 Adjacent and hypotenuse: cos	$AC = 15\cos 40° = 11.5$ cm

APPLY

1 Hypotenuse 20, angle 25°: the side opposite the angle.

2 Opposite 7, adjacent 4: the angle.

I.2 The sine rule

Property 1 *the sine rule, the cosine rule, the area*

In any triangle ABC, side a facing A, and so on:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}, \qquad c^2 = a^2 + b^2 - 2ab\cos C, \qquad \text{Area} = \frac{1}{2}ab\sin C.$$

Example 2

In the triangle ABC, $\hat{A} = 48°$, $\hat{B} = 71°$ and $a = 12$. Find b.

WHAT YOU DO	WHAT YOU WRITE
Step 1 A complete pair: a and $\hat{A}$	$\dfrac{12}{\sin 48°}$
Step 2 Write the sine rule	$\dfrac{b}{\sin 71°} = \dfrac{12}{\sin 48°}$
Step 3 Solve, calculator in degrees	$b = 15.3$

APPLY

3 $a = 9$, $\hat{A} = 30°$, $\hat{B} = 80°$: find b.

4 $a = 8$, $\hat{A} = 40°$, $b = 6$: find $\hat{B}$.

I.3 The cosine rule: a side

Watch out 1

The sine rule needs a complete pair: a side and the angle opposite it. Two sides and the angle between them: the cosine rule. In Paper 2, the calculator in degrees.

Example 3

In the triangle ABC, $a = 8$, $b = 11$ and $\hat{C} = 72°$. Find c.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Two sides and the angle between them: cosine rule	$c^2 = 8^2 + 11^2 - 2 \times 8 \times 11\cos 72°$
Step 2 Compute c^2	$c^2 = 130.61\ldots$
Step 3 Square root	$c = 11.4$

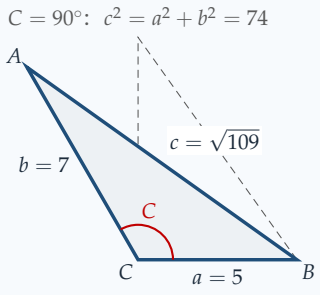


Figure 1 — $a = 5$, $b = 7$ and the angle C between them: at $C = 90°$, $c^2 = a^2 + b^2$; the cosine rule corrects Pythagoras by $-2ab\cos C$.

APPLY

5 $b = 5$, $c = 9$, $\hat{A} = 110°$: find a.

6 $a = 7$, $b = 15$, $\hat{C} = 60°$: find c exactly.

I.4 The cosine rule: an angle

Example 4

A triangle has sides 6, 9 and 13. Find its largest angle.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The largest angle faces the longest side	the angle C opposite 13
Step 2 $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$	$\cos C = \dfrac{36 + 81 - 169}{108} = -0.481\ldots$
Step 3 Inverse cosine: obtuse is fine	$C = 118.8°$

APPLY

7 Sides 4, 5, 6: the largest angle.

8 Sides 7, 8, 9: the smallest angle.

I.5 The area of a triangle

Example 5

Find the area of the triangle with $a = 10$, $b = 13$ and $\hat{C} = 38°$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Two sides and the angle between them	$\frac{1}{2}ab\sin C$
Step 2 Substitute	$\frac{1}{2} \times 10 \times 13 \times \sin 38°$
Step 3 Compute	40.0

APPLY

9 Sides 12 and 7, angle 140° between them: the area.

10 Area 25, sides 8 and 10: the acute angle between them.

II BEARINGS AND ELEVATION

SL 3.3 · bearings, angles of elevation and depression, labelled diagrams

II.1 Bearings

Definition 2 *bearing, elevation, depression*

A **bearing** is measured **clockwise from north**, with three figures: 060°. The angles of **elevation** and **depression** are measured from the **horizontal**.

Property 2 *north lines are parallel*

The north lines at two points are parallel. The bearing of A from B is the bearing of B from A plus or minus 180°.

Example 6

A ship sails 8 km on a bearing of 040°. How far north and how far east has it gone? What is the bearing back to its start?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Draw north; the angle is from north	right-angled triangle, angle 40° at the start
Step 2 North: adjacent; east: opposite	$8\cos 40^\circ = 6.13$ km, $8\sin 40^\circ = 5.14$ km
Step 3 Back bearing: $+180^\circ$	220°

APPLY

1 The bearing of A from B is 300°: the bearing of B from A.

2 15 km on a bearing of 210°: how far south and west?

II.2 A journey in two legs

Watch out 2

Draw the north line at every point you turn at. The angle of depression from T to A equals the angle of elevation from A to T.

Example 7

A boat sails 7 km from P on a bearing of 030° to A, then 9 km on a bearing of 110° to B. Find PB.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Back bearing at A	bearing of P from A: 210°
Step 2 The angle at A	$210^\circ - 110^\circ = 100^\circ$
Step 3 Cosine rule	$PB^2 = 7^2 + 9^2 - 2\times 7\times 9\cos 100^\circ = 151.88\ldots$
Step 4 Square root	$PB = 12.3$ km

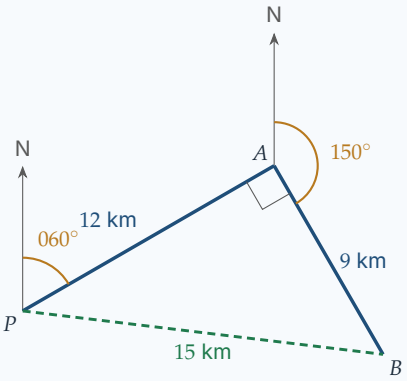


Figure 2 — Exam-style question 3's journey: 060° then 150°, a right angle at A, $PB = 15$ km.

APPLY

3 5 km on 090°, then 12 km on 180°: the distance and the bearing of the end from the start.

II.3 Angles of elevation and depression

Example 8

From a point 50 m from the foot of a tree, the angle of elevation of its top is 28°. Find the height of the tree.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Draw the horizontal, the angle from it	right angle at the foot of the tree
Step 2 Opposite and adjacent: tan	$\tan 28^\circ = \dfrac{h}{50}$
Step 3 Solve	$h = 26.6$ m

APPLY

4 From the top of a 60 m cliff, the angle of depression of a boat is 18°: its distance from the foot of the cliff.

5 A kite string 40 m long makes 55° with the ground: the height of the kite.

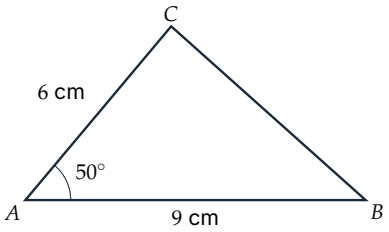
III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Solving a triangle

EXAM-STYLE QUESTION 1 Paper 2 · calculator · [Maximum mark: 6]

In the triangle ABC, $AB = 9$ cm, $AC = 6$ cm and $B\hat{A}C = 50°$.



(a) Find BC. [2]

(b) Find the area of the triangle ABC. [2]

(c) Find $A\hat{B}C$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Two sides and the angle between them: cosine rule	$BC^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \cos 50°$	M1
Step 2 Square root	$BC = 6.90$ cm $(6.89774\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\frac{1}{2}ab\sin C$	$\frac{1}{2} \times 9 \times 6 \sin 50°$	(M1)
Step 2 Compute	20.7 cm^2 $(20.6831\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Sine rule with the side just found	$\frac{\sin B}{6} = \frac{\sin 50°}{6.89774\ldots}$	M1
	Use the unrounded value of BC.	
Step 2 Inverse sine	$A\hat{B}C = 41.8°$ $(41.7854\ldots)$	A1

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 5]

In the triangle PQR, $PQ = 4$, $PR = 6$ and $Q\hat{P}R = 60°$.

(a) Find QR, giving your answer in the form $k\sqrt{7}$. [3]

(b) Find the exact area of the triangle PQR. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Cosine rule	$QR^2 = 4^2 + 6^2 - 2 \times 4 \times 6\cos 60°$	M1
Step 2 $\cos 60° = \frac{1}{2}$	$= 52 - 48 \times \frac{1}{2} = 28$	A1
	$\cos 60° = \frac{1}{2}$.	
Step 3 Simplify the root	$QR = 2\sqrt{7}$	A1

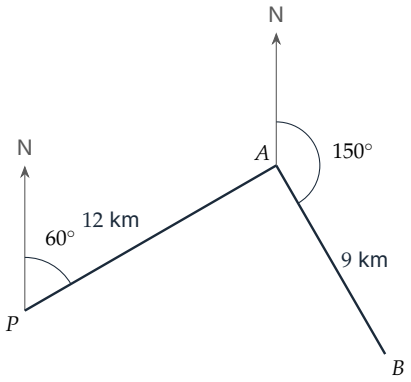
WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\frac{1}{2}ab\sin C$, $\sin 60° = \frac{\sqrt{3}}{2}$	$\frac{1}{2} \times 4 \times 6 \times \frac{\sqrt{3}}{2}$	M1
	$\sin 60° = \frac{\sqrt{3}}{2}$.	
Step 2 Simplify	$6\sqrt{3}$	A1

B Bearings (Paper 2)

EXAM-STYLE QUESTION 3 Paper 2 · calculator · [Maximum mark: 7]

A ship sails from a port P on a bearing of 060° for 12 km to a point A. It then sails on a bearing of 150° for 9 km to a point B.



(a) Show that $P\hat{A}B = 90°$. [2]

(b) Find the distance PB. [2]

(c) Find the bearing of B from P. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Back bearing at A	the bearing of P from A is $060° + 180° = 240°$	M1
	Or: co-interior angles on the north lines, $180° - 60° = 120°$, then $120° - 30°$.	
Step 2 Subtract the bearings	$240° - 150°$	A1
Step 3 Conclude	$P\hat{A}B = 90°$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Pythagoras	$PB^2 = 12^2 + 9^2$	(M1)
Step 2 Compute	$PB = 15$ km	A1

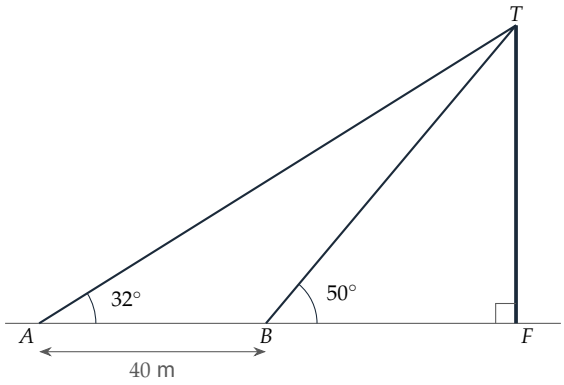
WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 tan of the angle at P	$\tan A\hat{P}B = \dfrac{9}{12}$	(M1)
Step 2 Compute the angle	$A\hat{P}B = 36.8699\ldots°$	(A1)
Step 3 Add to the first bearing	$060° + 36.87° = 096.9°$	A1
	Add the bearing of A: B is clockwise from A.	

C Angles of elevation (Paper 2)

EXAM-STYLE QUESTION 4 Paper 2 · calculator · [Maximum mark: 5]

From a point A on level ground, the angle of elevation of the top T of a tower is $32°$. From a point B, 40 m closer to the tower on the same straight line, the angle of elevation is $50°$.



(a) Find $A\hat{T}B$. [1]

(b) Find BT. [2]

(c) Find the height of the tower. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Exterior angle of triangle ABT	$50° - 32° = 18°$	A1
	The exterior angle of triangle ABT at B is $50°$.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Sine rule in triangle ABT	$\dfrac{BT}{\sin 32°} = \dfrac{40}{\sin 18°}$	M1
Step 2 Compute	$BT = 68.6$ m $(68.5941\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Right-angled triangle at the foot: sin	$h = 68.594\ldots \times \sin 50°$	(M1)
	The right-angled triangle at the foot of the tower.	
Step 2 Compute	$h = 52.5$ m $(52.5461\ldots)$	A1

IV WHICH METHOD? READ THE QUESTION

3.3 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
a right angle in the triangle	SOH CAH TOA	the ratio with the two sides involved D1
a side and its opposite angle, and one more	the sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B}$ P1
two sides and the angle between them	the cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$ P1
three sides, an angle wanted	the cosine rule, rearranged	$\cos C = \frac{a^2+b^2-c^2}{2ab}$ P1
"find the area"	$\frac{1}{2}ab\sin C$	two sides and the angle between them P1
"on a bearing of"	clockwise from north	a north line at each point; back bearing $\pm 180°$ P2
"angle of elevation", "of depression"	from the horizontal	a right-angled triangle, often two of them D2

In the exam 1 *command terms*

Find: the rule with the numbers in it, then the answer to 3 s.f., angles to 1 decimal place. **Show that**: every step, ending on the given value. **Exact**: surds, no decimals.

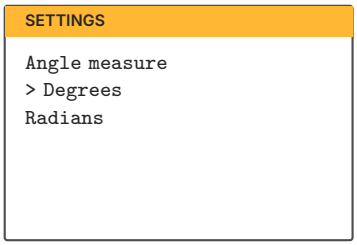
V ON THE GDC: PAPER 2

SL 3.2 · 3.3 · NumWorks · every calculator move of this topic

1 Degrees first

Settings → Angle measure → Degrees

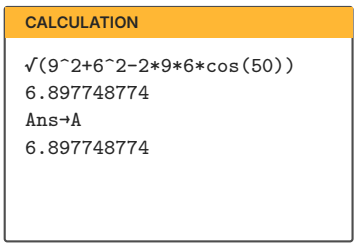
Every angle of this topic is in degrees. A wrong mode gives a wrong answer that looks plausible: check the banner shows deg.



2 The cosine rule in one line

Calculation → √(9^2+6^2−2×9×6×cos(50)) → EXE → sto→ A

Type the whole rule, then store the result in A: the next steps use the full value. **On paper:** the rule with its numbers, then $BC = 6.90$ cm.



3 An angle from the sine rule

Calculation → shift → sin → 6×sin(50)/A → EXE

shift sin gives arcsin, with A the stored side. **On paper:** "$\frac{\sin B}{6} = \frac{\sin 50^\circ}{6.8977}$, $B = 41.8^\circ$".

CALCULATION

```
arcsin(6*sin(50)/A)
41.78543988
```

ANSWERS TO THE APPLY QUESTIONS

I.1 8.45

I.2 60.3°

I.3 17.7

I.4 28.8°

I.5 11.7

I.6 $c^2 = 169$: 13

I.7 82.8°

I.8 48.2°

I.9 27.0

I.10 $\sin C = 0.625$: 38.7°

II.1 120°

II.2 13.0 km; 7.5 km

II.3 13 km; 157.4°

II.4 185 m

II.5 32.8 m

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