# Trigonometric equations

Skills sheet · SL AA 3.8

## KEY FACTS

arcsin, arccos, arctan: one value only D1
$\sin x = k$: $\alpha$, $\pi - \alpha$; $\cos x = k$: $\pm\alpha$; tan: $+\pi$ P1
$\sin(bx) = k$: the interval times $b$ P2
An identity, then a quadratic in $\sin x$ or $\cos x$ P3

*Stretch the interval with $bx$* W1.
*Never divide by $\sin x$* W2.
*Reject $|\sin x| > 1$, and say so.*

## WHAT UNLOCKS

- $\sin x = k \rightarrow$ two solutions per period, symmetric about $\frac{\pi}{2}$.
- $\sin 2x = k \rightarrow$ $2x$ over $[0, 4\pi]$, then halve.
- $\sin^2 x$ with $\cos x \rightarrow$ $\sin^2 x = 1 - \cos^2 x$, then factorise.
- no algebra possible $\rightarrow$ graphs on the GDC.

## WORKED EXAMPLE ①

**Solve** $\sin x = -\frac{1}{2}$, $0 \leq x \leq 2\pi$. no GDC

$\alpha = \frac{\pi}{6}$, third and fourth quadrants — M1
$\frac{7\pi}{6}$, $\frac{11\pi}{6}$ — A1A1

**Method**
*A sine equation*
① The reference angle from $|k|$.
② The quadrants from the sign.
③ Every solution in the interval.

## WORKED EXAMPLE ②

**Solve** $\cos x = 0.3$, $0 \leq x \leq 2\pi$. with GDC

$x = 1.27$ — A1
$x = 2\pi - 1.27 = 5.02$ — A1

**Method**
*A cosine equation*
① arccos on the GDC, radians.
② The other: $2\pi - \alpha$.
③ 3 s.f.
***Not $\pi - \alpha$ for a cosine.***

## FLUENCY

**1.** $\sin x = 1$ no GDC
**2.** $\cos x = 0$ no GDC
**3.** $\tan x = 1$ no GDC
**4.** $\cos x = -1$ no GDC
**5.** $\sin x = \frac{\sqrt{3}}{2}$ no GDC
**6.** $\sin x = 2$ no GDC

## REASONING

●●● **Question 1.** Solve $\cos 2x = 0, 0 \leq x \leq \pi$. [no GDC]

(a) Find every solution. [3]

●●● **Question 2.** Solve $\tan^2 x = 3, 0 \leq x \leq \pi$. [no GDC]

(a) Find every solution. [3]

●●● **Question 3.** Solve $2\sin x = x, 0 \leq x \leq \pi$. [with GDC]

(a) Find every solution. [2]

## PROBLEMS

●●● **Question 4.** Solve $\cos 2x = \sin x, 0 \leq x \leq 2\pi$. [no GDC]

(a) Find every solution. [6]

●●● **Question 5.** Solve $\sin 2x = \sin x, 0 \leq x \leq 2\pi$. [no GDC]

(a) Find every solution. [6]

## ANSWERS

**1** $\frac{\pi}{2}$

**2** $\frac{\pi}{2}, \frac{3\pi}{2}$

**3** $\frac{\pi}{4}, \frac{5\pi}{4}$

**4** $\pi$

**5** $\frac{\pi}{3}, \frac{2\pi}{3}$

**6** no solution

**Q1** $\frac{\pi}{4}, \frac{3\pi}{4}$

**Q2** $\frac{\pi}{3}, \frac{2\pi}{3}$

**Q3** 0 and 1.90

**Q4** $(2\sin x - 1)(\sin x + 1) = 0$: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

**Q5** $\sin x(2\cos x - 1) = 0$: $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

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