Trigonometric functions

Tides, Ferris wheels, daylight hours: whatever repeats is modelled by a sine or a cosine. Four numbers describe such a graph completely — how tall the wave is, how long it takes, where it is centred and where it starts — and every question asks for one of them.

I THE GRAPHS OF SIN, COS AND TAN

SL 3.7 · period, amplitude, $y = a\sin\big(b(x-c)\big)+d$

I.1 Period and amplitude of sin, cos, tan

Definition 1 *period and amplitude*

Periodic with period p: $f(x+p)=f(x)$. $\sin x$ and $\cos x$: period 2π, range $-1 \le y \le 1$, amplitude 1. $\tan x$: period π, vertical asymptotes $x = \frac{\pi}{2} + k\pi$.

Example 1

For $y = 4\cos x$, $0 \le x \le 2\pi$, write down the amplitude, the period, the range and the maximum points.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Amplitude: the factor in front	4
Step 2 Period: the same as $\cos x$	2π
Step 3 Range	$-4 \le y \le 4$
Step 4 Maximum where $\cos x = 1$	$(0, 4)$ and $(2\pi, 4)$

APPLY

1 $y = 5\sin x$: amplitude and range.

2 $y = \tan x$, $0 \le x \le 2\pi$: the asymptotes.

I.2 The period $\frac{2\pi}{b}$

Property 1 *the four parameters*

$y = a\sin\big(b(x-c)\big) + d$ (or cos): amplitude $|a|$, period $\dfrac{2\pi}{b}$, horizontal shift c, principal axis $y = d$; maximum $d + |a|$, minimum $d - |a|$.

Watch out 1

The period is $\frac{2\pi}{b}$, not b: a larger b squeezes the graph.

Example 2

Find the period of $y = \cos 3x$ and of $y = \sin\dfrac{x}{2}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Read b	$b = 3$ and $b = \frac{1}{2}$
Step 2 Period $\dfrac{2\pi}{b}$	$\dfrac{2\pi}{3}$ and $\dfrac{2\pi}{1/2} = 4\pi$

APPLY

3 Period of $\sin 5x$.

4 Period of $\cos\frac{\pi x}{4}$.

5 Period of $\tan 3x$.

I.3 The range: the axis and the amplitude

Example 3

For $y = 2\cos 4x - 3$, write down the amplitude, the principal axis, the range and the period.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Amplitude $\lvert a\rvert$, axis $y = d$	2; $y = -3$
Step 2 Range: $d - \lvert a\rvert$ to $d + \lvert a\rvert$	$-5 \le y \le -1$
The axis moves the whole wave.	
Step 3 Period	$\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

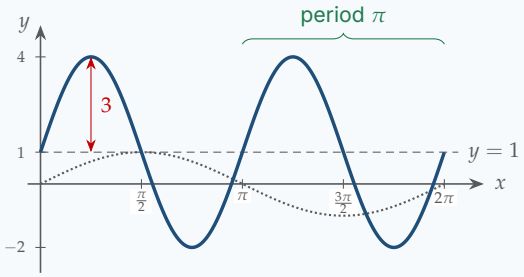


Figure 1 — Exam-style question 1: $y = 3\sin 2x + 1$, amplitude 3 around $y = 1$, range $-2 \le y \le 4$, period π.

APPLY

6 Range of $y = 6\sin x + 2$.

7 Range of $y = -3\cos 2x + 5$.

I.4 Maximum and minimum points

Example 4

Find the maximum and minimum points of $y = 2\sin 3x + 1$, for $0 \leq x \leq \dfrac{2\pi}{3}$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Maximum: the inside is $\dfrac{\pi}{2}$	$3x = \dfrac{\pi}{2}$: $\left(\dfrac{\pi}{6}, 3\right)$
Step 2 Minimum: the inside is $\dfrac{3\pi}{2}$	$3x = \dfrac{3\pi}{2}$: $\left(\dfrac{\pi}{2}, -1\right)$
Step 3 Check: $d \pm \lvert a \rvert$	$1 + 2 = 3,\ 1 - 2 = -1$

APPLY

8 $y = 5\cos 2x - 1, 0 \leq x \leq \pi$: the minimum point.

9 $y = 5\cos 2x - 1, 0 \leq x \leq \pi$: the maximum points.

I.5 A horizontal shift

Example 5

Describe the graph of $y = \sin\left(x - \dfrac{\pi}{3}\right)$ and find its first maximum for $x \geq 0$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Read c	$c = \dfrac{\pi}{3}$: a shift $\dfrac{\pi}{3}$ to the right
Step 2 The maximum of sin moves too	$x = \dfrac{\pi}{2} + \dfrac{\pi}{3} = \dfrac{5\pi}{6}$: $\left(\dfrac{5\pi}{6}, 1\right)$

APPLY

10 $y = \cos\left(x + \frac{\pi}{4}\right)$: the first maximum for $x \geq 0$.

II.1 a, b and d from a context

Property 2 *the parameters from the context*

$a = \dfrac{\max - \min}{2}$, $d = \dfrac{\max + \min}{2}$, $b = \dfrac{2\pi}{\text{period}}$. A model that starts at its maximum is a cos; one that starts on its axis going up is a sin.

Watch out 2

A minimum at $t = 6$ after a maximum at $t = 0$ means half a period is 6: the period is 12. Check the model at one known point.

Example 6

The temperature in a town is highest, 24 °C, at 14:00 and lowest, 10 °C, at 02:00. It is modelled by $T = a\cos\big(b(t-14)\big) + d$, t in hours. Find a, b and d.

WHAT YOU DO	WHAT YOU WRITE
Step 1 a: half the difference	$a = \dfrac{24-10}{2} = 7$
Step 2 d: half the sum	$d = \dfrac{24+10}{2} = 17$
Step 3 Period: max to min is half of it	$12\text{ h} \times 2 = 24\text{ h}$, $b = \dfrac{2\pi}{24} = \dfrac{\pi}{12}$

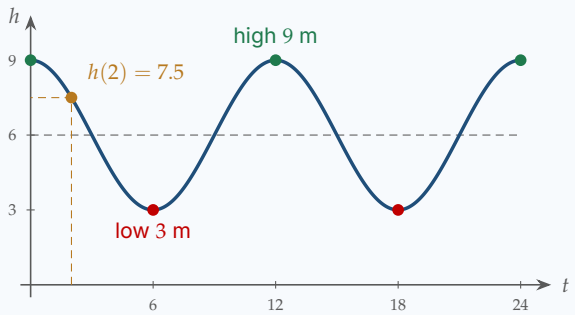


Figure 2 — Exam-style question 2: $h(t) = 3\cos\frac{\pi t}{6} + 6$, high tide 9 m at midnight, low tide 3 m at 6:00.

APPLY

1 A wheel: highest point 35 m, lowest 3 m, one turn every 8 minutes. a, b, d?

2 High water 7 m, the next low water 1 m, 6.2 hours later. a, b, d?

II.2 Using a model on the GDC

Example 7

The depth of water is $D(t) = 8 - 3\cos\dfrac{\pi t}{5}$ metres, $0 \leq t \leq 10$. Find $D(3)$, and for how long the depth is more than 9 m.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Substitute, calculator in radians	$D(3) = 8.93$ m
Step 2 Intersections of $y = D(t)$ and $y = 9$	$t = 3.04$ and $t = 6.96$
Step 3 Subtract the two times	3.92 hours

APPLY

3 Same model: $D(1)$.

4 Same model: the least depth, and when.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Reading a trigonometric function

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 6]

Let $f(x) = 3\sin(2x) + 1$, for $0 \leq x \leq \pi$.

(a) Write down the amplitude of f. [1]

(b) Find the period of f. [1]

(c) Write down the range of f. [2]

(d) Find the coordinates of the maximum point of the graph of f. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Amplitude $\lvert a \rvert$	3	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Period $\dfrac{2\pi}{b}$	$\dfrac{2\pi}{2} = \pi$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Range: $d - \lvert a \rvert$ to $d + \lvert a \rvert$	$-2 \leq f(x) \leq 4$ *A1 for each end point; the inequality, not just two numbers.*	A1A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Maximum: the inside is $\dfrac{\pi}{2}$	$2x = \dfrac{\pi}{2},\ x = \dfrac{\pi}{4}$	A1
Step 2 $y = d + \lvert a \rvert$	$\left(\dfrac{\pi}{4},\ 4\right)$	A1

B A model from its high and low

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 6]

The depth of water in a harbour, h metres, t hours after midnight, is modelled by $h(t) = a\cos(bt) + d$. High tide, 9 m, is at midnight; the next low tide, 3 m, is at 6:00.

(a) Find the values of a and d. [2]

(b) Find the value of b. [2]

(c) Find the depth of water at 2:00. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 a: half the difference	$a = \dfrac{9-3}{2} = 3$	A1
Step 2 d: half the sum	$d = \dfrac{9+3}{2} = 6$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 High to low: half a period	$\text{period} = 12$ *From a high tide to the next low tide is half a period.*	(A1)
Step 2 $b = \dfrac{2\pi}{\text{period}}$	$b = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute $t = 2$	$h(2) = 3\cos\dfrac{\pi}{3} + 6$	(M1)
Step 2 Compute	$= 7.5$ m $\cos\frac{\pi}{3} = \frac{1}{2}$.	A1

C With the GDC (Paper 2)

EXAM-STYLE QUESTION 3 Paper 2 · calculator · [Maximum mark: 6]

The height, h metres, of a seat on a Ferris wheel t minutes after it starts is $h(t) = 20 - 18\cos\left(\frac{\pi t}{15}\right)$, for $t \geq 0$.

(a) Write down the greatest height of the seat. [1]

(b) Find the time for one revolution. [1]

(c) Find the height of the seat when $t = 10$. [1]

(d) Find, for one revolution, the length of time during which the seat is higher than 30 m. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $d + \lvert a \rvert$	38 m	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Period $\frac{2\pi}{b}$	$\frac{2\pi}{\pi/15} = 30$ minutes	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute $t = 10$	29 m $\cos\frac{2\pi}{3} = -\frac{1}{2}$.	A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the equation	$20 - 18\cos\left(\frac{\pi t}{15}\right) = 30$ *Or the graph of h with the line h = 30.*	(M1)
Step 2 Intersections on the GDC	$t = 10.3124\ldots$ and $t = 19.6875\ldots$	(A1)
Step 3 Subtract the two times	9.38 minutes $(9.37516\ldots)$	A1

EXAM-STYLE QUESTION 4 Paper 2 · calculator · [Maximum mark: 6]

Let $f(x) = 2\sin x + \cos 2x$, for $0 \leq x \leq 2\pi$.

(a) Find the range of f. [3]

(b) Solve $f(x) = 0$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Maximum on the GDC	maximum 1.5	A1
	At $x = \frac{\pi}{6}$ *and* $x = \frac{5\pi}{6}$.	
Step 2 Minimum on the GDC	minimum -3	A1
	At $x = \frac{3\pi}{2}$.	
Step 3 Write the range	$-3 \leq f(x) \leq 1.5$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Graph f on the GDC	a graph of f and its zeros	(M1)
	Or $\cos 2x = 1 - 2\sin^2 x$ *and a quadratic in* $\sin x$.	
Step 2 Zeros	$x = 3.52$ $(3.51632\ldots)$	A1
	$x = 5.91$ $(5.90845\ldots)$	A1

IV WHICH METHOD? READ THE QUESTION

3.7 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"amplitude"	$\lvert a\rvert$	half the distance from max to min P1
"period"	$\frac{2\pi}{b}$	in degrees: $\frac{360^\circ}{b}$ P1
"range"	$d-\lvert a\rvert$ to $d+\lvert a\rvert$	an inequality in y P1
a maximum point	the inside equals $\frac{\pi}{2}$ (for sin)	solve for x, then $y=d+\lvert a\rvert$ P1
greatest and least values in a context	a and d	half the difference, half the sum P2
time from a high to the next low	half a period	$b=\frac{2\pi}{\text{period}}$ P2
"how long is it above k"	two times in one cycle	the GDC: intersections, then the difference

In the exam 1 *command terms*

Write down: read from the formula or the graph, no working. **Find**: show the formula with its numbers. **Hence**: use the previous answer. In a context, give the units.

V ON THE GDC: PAPER 2

SL 3.7 · NumWorks · every calculator move of this topic

1 Radians and a window of one period

Settings → Radians → Grapher → Axes → x: 0 to 30

Trigonometric functions are drawn in radians. Set x over one period and y from the minimum to the maximum.

GRAPHER · AXES

```
Angle measure: Radians
f(x)=20-18cos(πx/15)
X: 0 .. 30 Y: 0 .. 40
```

2 Where the graph crosses a level

Grapher → g(x)=30 → Find → Intersection

Add the line $y=30$, then find each intersection in the window. **On paper:** the equation, both times, then their difference.

GRAPHER · INTERSECTION

```
x=10.31242 y=30
x=19.68758 y=30
```

3 The greatest and least values

Grapher → Find → Maximum → Minimum

For a composite function such as $2\sin x + \cos 2x$, the GDC finds the extremes the formula cannot give at a glance.

GRAPHER · MAXIMUM

```
f(x)=2sin(x)+cos(2x)
x=0.5235988 y=1.5
x=4.712389 y=-3
```

ANSWERS TO THE APPLY QUESTIONS

I.1 5; $-5 \le y \le 5$

I.2 $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

I.3 $\frac{2\pi}{5}$

I.4 8

I.5 $\frac{\pi}{3}$

I.6 $-4 \le y \le 8$

I.7 $2 \le y \le 8$

I.8 $\left(\frac{\pi}{2}, -6\right)$

I.9 $(0, 4)$ and $(\pi, 4)$

I.10 $\left(\frac{7\pi}{4}, 1\right)$

II.1 16; $\frac{\pi}{4}$; 19

II.2 3; 0.507; 4

II.3 5.57 m

II.4 5 m, at $t = 0$ and $t = 10$

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