# Trigonometric functions



## KEY FACTS

sin, cos: period $2\pi$; tan: period $\pi$ D1

$a\sin\big(b(x-c)\big)+d$: amplitude $|a|$, period $\frac{2\pi}{b}$ P1

Max $d+|a|$, min $d-|a|$, axis $y=d$ P1

From a context: $a=\frac{\max-\min}{2}$, $d=\frac{\max+\min}{2}$ P2

*The period is $\frac{2\pi}{b}$, not $b$* W1.

*High to the next low: half a period* W2.

*Radians on the GDC.*

## WHAT UNLOCKS

- a formula $\rightarrow$ read $a$, $b$, $d$.
- a high and a low $\rightarrow$ $a$ and $d$; the time between them $\rightarrow$ $b$.
- starts at a maximum $\rightarrow$ a cosine.
- "how long above $k$" $\rightarrow$ two intersections on the GDC.

## WORKED EXAMPLE ①

**Find the period of $y=5\sin(4x)$.** no GDC

$\frac{2\pi}{4}$ M1

$=\frac{\pi}{2}$ A1

**Method**

*The period*

① Read $b$.
② $\frac{2\pi}{b}$.
③ Simplify.

*$b=4$ **squeezes the graph.***

## WORKED EXAMPLE ②

**A wave has maximum 10 and minimum 2. Find $a$ and $d$.** no GDC

$a=\frac{10-2}{2}=4$ A1

$d=\frac{10+2}{2}=6$ A1

**Method**

*Amplitude and axis*

① Half the difference.
② Half the sum.
③ Check: $d\pm a$.

## FLUENCY

**1.** Amplitude of $y=-2\cos x$ no GDC

**2.** Period of $y=\cos\frac{x}{3}$ no GDC

**3.** Range of $y=5\sin x-1$ no GDC

**4.** Period of $y=\tan 2x$ no GDC

**5.** Period of $y=\sin(30x)$, $x$ in degrees no GDC

**6.** Principal axis of $y=4\cos 2x+7$ no GDC

## REASONING

●●● **Question 1.** $y = a\sin(bx) + d$ has a maximum at $\left(\frac{\pi}{4}, 5\right)$ and the next minimum at $\left(\frac{3\pi}{4}, -1\right)$. [no GDC]

(a) Find $a$, $b$ and $d$. [4]

●●● **Question 2.** The hours of daylight are $D(t) = 12 + 4\sin\left(\frac{2\pi(t-80)}{365}\right)$, $t$ the day of the year. [no GDC]

(a) Find the greatest number of hours and the day it occurs. [3]

●●● **Question 3.** $f(x) = 3\sin x + 4$, $0 \leq x \leq 2\pi$. [no GDC]

(a) Find the minimum value and where it occurs. [2]

## PROBLEMS

●●● **Question 4.** $h(t) = 10 + 8\sin\left(\frac{\pi t}{6}\right)$, $0 \leq t \leq 12$. [no GDC]

(a) Find how long $h$ is above 14. [4]

●●● **Question 5.** $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$. [with GDC]

(a) Find the maximum value of $f$ and where it occurs. [2]

## ANSWERS

**1** 2

**2** $6\pi$

**3** $-6 \le y \le 4$

**4** $\frac{\pi}{2}$

**5** $12^\circ$

**6** $y = 7$

**Q1** $a = 3, b = 2, d = 2$

**Q2** 16 hours, day 171.25

**Q3** 1 at $x = \frac{3\pi}{2}$

**Q4** $\sin\frac{\pi t}{6} > \frac{1}{2}$: $1 < t < 5,\ 4$

**Q5** $\sqrt{2} = 1.41$ at $x = \frac{\pi}{4}$

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