Antiderivatives

Integration undoes differentiation: from the gradient function f', find f. There is never one answer but a family, $F(x)+c$, and the exam fixes c with a point of the curve. Three skills carry the whole topic: powers of x, the standard functions with a linear inside, and spotting the derivative of an inner function.

I ANTIDERIVATIVES AND THE CONSTANT

SL 5.5 · 5.10 · $\int x^n \, dx$, the boundary condition

I.1 The power rule backwards

Definition 1 *antiderivative, indefinite integral*

F is an **antiderivative** of f when $F'(x) = f(x)$. All of them: $\int f(x)\,dx = F(x) + c$, the **indefinite integral**.

Property 1 *powers, sums, multiples*

For $n \neq -1$: $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + c$ — add one to the power, divide by the new power. Term by term; a constant factor stays in front.

Example 1

Find $\int (6x^2 - 4x + 3)\,dx$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Each term: power up by one, divide by it	$\dfrac{6x^3}{3},\ -\dfrac{4x^2}{2},\ 3x$
Step 2 Simplify, add c	$2x^3 - 2x^2 + 3x + c$
Step 3 Check: differentiate	$6x^2 - 4x + 3$

APPLY

1 $\int \left(5x^4 + 1\right) dx$

2 $\int x^{-4}\,dx$

3 $\int (x+3)(x-1)\,dx$

I.2 Roots and fractions

Watch out 1

Roots and fractions are powers first: $\sqrt{x} = x^{\frac{1}{2}}$, $\dfrac{3}{x^2} = 3x^{-2}$. A product or a quotient is *not* integrated factor by factor: expand or divide first.

Example 2

Find $\displaystyle\int \left(\frac{2}{x^3} + 6\sqrt{x}\right) \mathrm{d}x$, for $x > 0$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Rewrite as powers	$2x^{-3} + 6x^{\frac{1}{2}}$
Step 2 Power up, divide by the new power	$\dfrac{2x^{-2}}{-2} + \dfrac{6x^{\frac{3}{2}}}{\frac{3}{2}}$
Step 3 Simplify, add c	$-x^{-2} + 4x^{\frac{3}{2}} + c$

APPLY

4 $\displaystyle\int \frac{1}{\sqrt{x}} \mathrm{d}x$

5 $\displaystyle\int \frac{x^2 + 4}{x^2} \mathrm{d}x$

I.3 The boundary condition

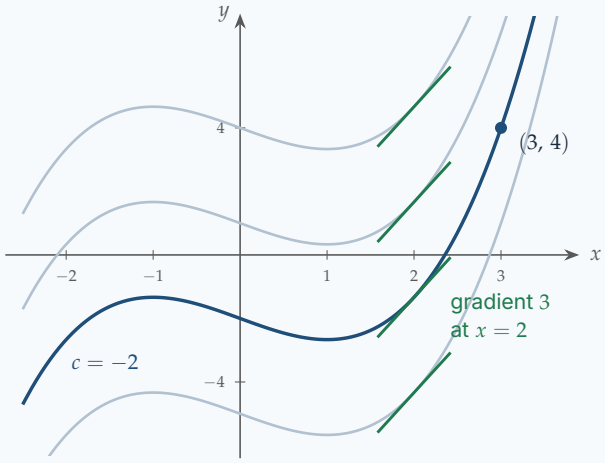


Figure 1 — The antiderivatives $\frac{x^3}{3} - x + c$ of $x^2 - 1$: one curve moved up or down; only one passes through $(3, 4)$, the one with $c = -2$.

Property 2 *the boundary condition*

From f' and one point (a, b): integrate with $+c$; write $f(a) = b$ and solve for c; write $f(x)$ in full.

Example 3

$f'(x) = 3x^2 - 4x$, and the graph of f passes through $(2, 5)$. Find $f(x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Integrate, with $+c$	$f(x) = x^3 - 2x^2 + c$
Step 2 Substitute the point	$8 - 8 + c = 5$: $c = 5$
Step 3 Write $f(x)$ in full	$f(x) = x^3 - 2x^2 + 5$

APPLY

6 $f'(x) = 4x + 1$, $f(0) = -3$: find $f(x)$.

7 $f'(x) = 2 - \dfrac{1}{x^2}$, $f(1) = 4$: find $f(x)$.

II STANDARD INTEGRALS AND A LINEAR INSIDE

SL 5.10 · e^x, $\frac{1}{x}$, $\sin x$, $\cos x$, and $f(ax+b)$

II.1 e^x, $\frac{1}{x}$, $\sin x$, $\cos x$

Property 3 *standard integrals*

$$\int e^x\,dx = e^x + c,\quad \int \frac{1}{x}\,dx = \ln x + c\ (x > 0),\quad \int \sin x\,dx = -\cos x + c,\quad \int \cos x\,dx = \sin x + c.$$

Example 4

Find $\int \left(e^x + \frac{3}{x} - 2\sin x\right) dx$, for $x > 0$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Each standard integral, factor kept	$e^x,\ 3\ln x,\ -2\times(-\cos x)$
Step 2 Simplify, add c	$e^x + 3\ln x + 2\cos x + c$

APPLY

1 $\int (\cos x + 4)\,dx$

2 $\int \frac{2}{x}\,dx,\ x > 0$

II.2 A linear inside: divide by a

Property 4 *a linear inside*

If $\int f(x)\,dx = F(x) + c$, then $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c$: $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$, $\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c$, $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b) + c$.

Watch out 2

Divide by a, never multiply. The minus sign belongs to $\int \sin x = -\cos x$. Differentiate the answer to check.

Example 5

Find $\int e^{3x+1}\,dx$ and $\int (4x-1)^3\,dx$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Integrate the outside	e^{3x+1} and $\frac{(4x-1)^4}{4}$
Step 2 Divide by a	$\div 3$ and $\div 4$
Step 3 Add c	$\frac{1}{3}e^{3x+1} + c$ and $\frac{(4x-1)^4}{16} + c$

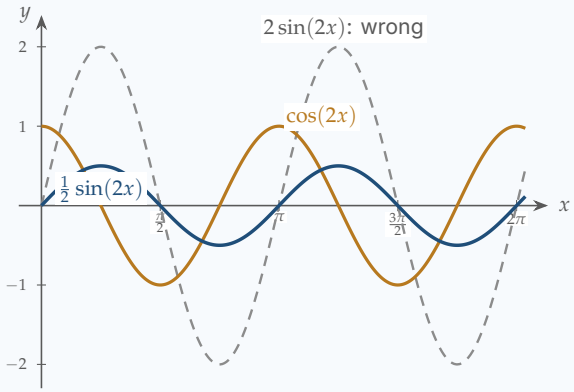


Figure 2 — $\cos 2x$ and its antiderivative $\frac{1}{2}\sin 2x$, half as tall; the dashed $2\sin 2x$ is what multiplying would give.

APPLY

3 $\displaystyle\int \cos 5x \, \mathrm{d}x$

4 $\displaystyle\int \frac{4}{4x-7} \, \mathrm{d}x$, $x > \frac{7}{4}$

5 $\displaystyle\int \sin\frac{x}{2} \, \mathrm{d}x$

II.3 A curve with exponentials or trigonometry

Example 6

$f'(x) = 4\cos 2x$ and $f(0) = 1$. Find $f(x)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Integrate, divide by 2	$f(x) = 2\sin 2x + c$
Step 2 Substitute: $\sin 0 = 0$	$c = 1$
Step 3 In full	$f(x) = 2\sin 2x + 1$

APPLY

6 $f'(x) = \mathrm{e}^{2x}$, $f(0) = 3$: find $f(x)$.

III INTEGRATION BY INSPECTION

SL 5.10 · $\int f(g(x))\,g'(x)\,\mathrm{d}x$

III.1 The chain rule backwards

Property 5 *the chain rule backwards*

$$\int f'(g(x))\,g'(x)\,\mathrm{d}x = f(g(x)) + c\text{; in particular } \int \frac{g'(x)}{g(x)}\,\mathrm{d}x = \ln g(x) + c \ (g > 0)$$
and $\displaystyle\int g'(x)\,\mathrm{e}^{g(x)}\,\mathrm{d}x = \mathrm{e}^{g(x)} + c.$

Example 7

Find $\displaystyle\int \frac{6x}{3x^2+2}\,\mathrm{d}x.$

WHAT YOU DO	WHAT YOU WRITE
Step 1 Is the top the derivative of the bottom?	$(3x^2+2)' = 6x$: yes
Step 2 $\frac{g'}{g}$: a logarithm	$\ln(3x^2+2) + c$

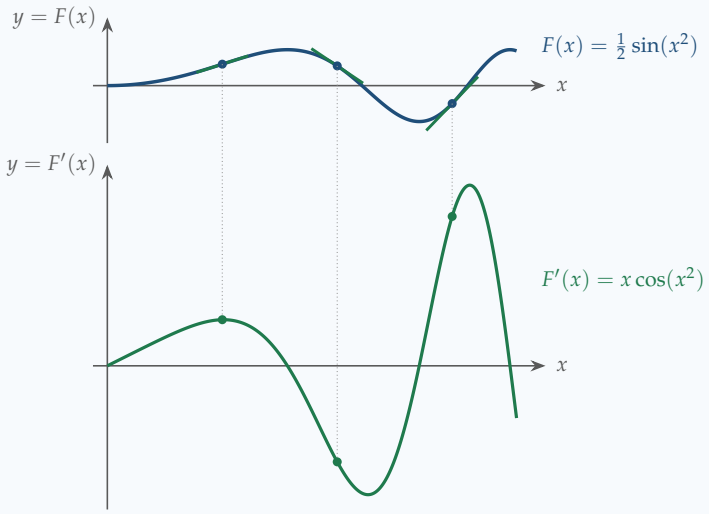


Figure 3 — $F(x) = \frac{1}{2}\sin x^2$ and its gradient $x\cos x^2$: the factor x is the derivative of the inside, halved.

APPLY

1 $\displaystyle\int \cos x\,\mathrm{e}^{\sin x}\,\mathrm{d}x$

2 $\displaystyle\int \frac{2x+1}{x^2+x+5}\,\mathrm{d}x$

III.2 Substitution, a constant factor adjusted

Property 6 *substitution*

Put $u = g(x)$, $\mathrm{d}u = g'(x)\,\mathrm{d}x$; integrate in u; write the answer back in x. A missing constant factor is adjusted.

Watch out 3

A missing *constant* can be adjusted; a missing x cannot: $\int \mathrm{e}^{x^2}\,\mathrm{d}x$ has no answer in standard functions.

Example 8

Find $\displaystyle\int x\,(x^2+1)^3\,\mathrm{d}x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Name the inside	$u = x^2+1,\ \mathrm{d}u = 2x\,\mathrm{d}x$
Step 2 Adjust the constant	$\dfrac{1}{2}\displaystyle\int u^3\,\mathrm{d}u$
Step 3 Integrate, back in x	$\dfrac{(x^2+1)^4}{8} + c$

APPLY

3 $\displaystyle\int x^2\mathrm{e}^{x^3}\,\mathrm{d}x$

4 $\displaystyle\int \cos x \sin^3 x\,\mathrm{d}x$

III.3 "Hence", after a derivative

Example 9

Differentiate $x\,\mathrm{e}^x$. Hence find $\displaystyle\int x\,\mathrm{e}^x\,\mathrm{d}x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Product rule	$(x\mathrm{e}^x)' = \mathrm{e}^x + x\mathrm{e}^x$
Step 2 Integrate both sides	$x\mathrm{e}^x = \displaystyle\int \mathrm{e}^x\,\mathrm{d}x + \int x\mathrm{e}^x\,\mathrm{d}x$
Step 3 Rearrange	$\displaystyle\int x\mathrm{e}^x\,\mathrm{d}x = x\mathrm{e}^x - \mathrm{e}^x + c$

APPLY

5 Differentiate $x\sin x$; hence find $\displaystyle\int x\cos x\,\mathrm{d}x$.

III.4 No antiderivative: the GDC

Example 10

$f'(x) = \cos\left(x^2\right)$ and $f(0) = 2$. Find $f(1)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $f(b) = f(a) + \int_a^b f'(x)\,\mathrm{d}x$	$f(1) = 2 + \int_0^1 \cos\left(x^2\right)\,\mathrm{d}x$
Step 2 The integral on the GDC	$0.904524\ldots$
Step 3 Add	$f(1) = 2.90$

APPLY

6 $g'(x) = \mathrm{e}^{-x^2}$, $g(0) = 0$: find $g(1)$.

IV EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Powers and the constant

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 5]

The gradient of a curve $y = f(x)$ is given by $f'(x) = 6x^2 - 4x + 1$. The curve passes through the point $(1, 5)$. Find $f(x)$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Integrate, with $+c$	$f(x) = 2x^3 - 2x^2 + x + c$	A1A1
	A1 for two correct terms, A1 for all three. The $+c$ is needed for what follows.	
Step 2 Substitute the point	$2(1)^3 - 2(1)^2 + 1 + c = 5$	M1
	Substitute the point into f, not into f'.	
Step 3 Solve for c	$c = 4$	A1
Step 4 In full	$f(x) = 2x^3 - 2x^2 + x + 4$	A1
	The function in full: "$c = 4$" alone is not the answer.	

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 4]

Find $\int \left(\frac{3}{x^2} + 4\sqrt{x}\right) \mathrm{d}x$, for $x > 0$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Rewrite as powers	$\int \left(3x^{-2} + 4x^{\frac{1}{2}}\right) \mathrm{d}x$	(M1)
	Every term as a power of x first.	
Step 2 First term	$-3x^{-1}$	A1
Step 3 Second term	$\frac{8}{3}x^{\frac{3}{2}}$	A1
	$4 \times \frac{x^{3/2}}{3/2}$: *dividing by $\frac{3}{2}$ multiplies by $\frac{2}{3}$.*	
Step 4 Simplify, add c	$-\frac{3}{x} + \frac{8}{3}x^{\frac{3}{2}} + c$	A1
	The whole answer, with its constant.	

B Standard integrals

EXAM-STYLE QUESTION 3

Paper 1 · no calculator · [Maximum mark: 6]

Find the following integrals.

(a) $\int (2x+1)^5 \, dx$ [2]

(b) $\int \frac{6}{3x-2} \, dx$, for $x > \frac{2}{3}$ [2]

(c) $\int 4\sin(2x) \, dx$ [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Outside, then divide by a	$\frac{(2x+1)^6}{12} + c$ *A1 for $(2x+1)^6$, A1 for dividing by $6 \times 2 = 12$.*	A1A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 A logarithm, divide by 3	$2\ln(3x-2) + c$ *A1 for $\ln(3x-2)$, A1 for $\frac{6}{3} = 2$.*	A1A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $-\cos$, divide by 2	$-2\cos(2x) + c$ *A1 for $-\cos(2x)$, A1 for $\frac{4}{2} = 2$.*	A1A1

EXAM-STYLE QUESTION 4

Paper 1 · no calculator · [Maximum mark: 5]

Let $f'(x) = e^{2x} + 3\cos x$. The graph of f passes through the point $(0, 4)$. Find $f(x)$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Divide by 2	$\frac{1}{2}e^{2x}$ *Divide by 2.*	A1
Step 2 $\int \cos = \sin$	$+3\sin x$	A1
Step 3 Substitute the point	$\frac{1}{2}e^0 + 3\sin 0 + c = 4$ $e^0 = 1$ *and* $\sin 0 = 0$.	M1
Step 4 Solve for c	$c = \frac{7}{2}$	A1
Step 5 In full	$f(x) = \frac{1}{2}e^{2x} + 3\sin x + \frac{7}{2}$	A1

C By inspection

EXAM-STYLE QUESTION 5 — Paper 1 · no calculator · [Maximum mark: 4]

Find $\int x\left(x^2-3\right)^4 \mathrm{d}x$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Name the inside	$u = x^2 - 3,\ \mathrm{d}u = 2x\,\mathrm{d}x$ *Or by inspection: try $(x^2-3)^5$ and differentiate it.*	(M1)
Step 2 Adjust the constant	$\frac{1}{2}\int u^4\,\mathrm{d}u$ *The x in front is half of $\mathrm{d}u$.*	A1
Step 3 Integrate, back in x	$\frac{1}{10}\left(x^2-3\right)^5 + c$ *A1 for $(x^2-3)^5$, A1 for $\frac{1}{10}$. Answer in x, not in u.*	A1A1

EXAM-STYLE QUESTION 6 — Paper 1 · no calculator · [Maximum mark: 5]

Let $f(x) = x\ln x$, for $x > 0$.

(a) Show that $f'(x) = \ln x + 1$. [2]

(b) Hence find $\int \ln x\,\mathrm{d}x$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Product rule	$f'(x) = 1 \times \ln x + x \times \frac{1}{x}$ *M1 for the product rule, A1 for both derivatives.*	M1A1
	$= \ln x + 1$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Integrate the derivative	$\int (\ln x + 1)\,\mathrm{d}x = x\ln x + c$ *Integrate the result of (a): it undoes the derivative.*	M1
Step 2 Rearrange	$\int \ln x\,\mathrm{d}x = x\ln x - x + c$ *A1 for $-x$ (the integral of 1), A1 for the whole answer with $+c$.*	A1A1

EXAM-STYLE QUESTION 7 Paper 1 · no calculator · [Maximum mark: 5]

Find the following integrals.

(a) $\displaystyle\int \frac{3x^2}{x^3+5}\,\mathrm{d}x$ [2]

(b) $\displaystyle\int \sin x \cos^4 x\,\mathrm{d}x$ [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Top: the derivative of the bottom	the numerator is the derivative of x^3+5	(M1)
	$\int \frac{g'}{g} = \ln g$.	
Step 2 A logarithm	$\ln\left(x^3+5\right)+c$	A1
	Valid where $x^3+5>0$.	

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Name the inside: $\cos x$	$u=\cos x,\ \mathrm{d}u=-\sin x\,\mathrm{d}x$	M1
	Or by inspection: the derivative of $\cos^5 x$ is $-5\sin x\cos^4 x$.	
Step 2 Integrate, back in x	$-\frac{1}{5}\cos^5 x+c$	A1A1
	A1 for $\cos^5 x$ with the minus sign, A1 for $\frac{1}{5}$.	

D With the GDC (Paper 2)

EXAM-STYLE QUESTION 8

Paper 2 · calculator · [Maximum mark: 5]

The function f is defined for $0 \leq x \leq 3$. Its derivative is $f'(x) = 3\sin\left(x^2\right)$, and $f(1) = 2$.

(a) Find $f(3)$. [3]

(b) Find the x-coordinate of the local maximum point of the graph of f. Justify your answer. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $f(3) = f(1) + \int_1^3 f'$	$f(3) = f(1) + \displaystyle\int_1^3 3\sin\left(x^2\right)\,\mathrm{d}x$	M1
	The change of f is the integral of f'.	
Step 2 The integral on the GDC	$= 2 + 1.38988\ldots$	(A1)
Step 3 Add	$f(3) = 3.39$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 f' from $+$ to $-$	$f'(x) = 0$ and f' changes from positive to negative	R1
	A maximum of f: f' crosses from $+$ to $-$.	
Step 2 Solve on the GDC	$x = 1.77$ $\left(\sqrt{\pi} = 1.77245\ldots\right)$	A1
	At $x = \sqrt{2\pi} = 2.51$, f' goes from $-$ to $+$: a minimum.	

EXAM-STYLE QUESTION 9

Paper 2 · calculator · [Maximum mark: 4]

A function g satisfies $g(0) = 1$ and $g'(x) = \mathrm{e}^{-0.5x^2}$.

(a) Write down the gradient of the graph of g at $x = 0$. [1]

(b) Find $g(2)$. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute $x = 0$	$g'(0) = 1$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $g(2) = g(0) + \int_0^2 g'$	$g(2) = 1 + \displaystyle\int_0^2 \mathrm{e}^{-0.5x^2}\,\mathrm{d}x$	M1
Step 2 The integral on the GDC	$= 1 + 1.19628\ldots$	(A1)
Step 3 Add	$g(2) = 2.20$	A1

V WHICH METHOD? READ THE QUESTION

5.10 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"find $\int$" with roots or fractions of x	powers of x	rewrite as x^n, then $\frac{x^{n+1}}{n+1}$, and $+c$ P1
"find $f(x)$", given $f'(x)$ and "passes through"	f' a point	integrate with $+c$; substitute the point; $f(x)$ in full P2
e^{ax+b}, $\sin(ax+b)$, $(ax+b)^n$	a linear inside	integrate the outside function, then divide by a P4
$\frac{1}{ax+b}$	a logarithm	$\frac{1}{a}\ln(ax+b)+c$ P4
a product with the derivative of the inside	$f(g(x))$ $g'(x)$, up to a constant	by inspection, or $u=g(x)$; the answer back in x P5
a fraction, the top the derivative of the bottom	$\frac{g'(x)}{g(x)}$	$\ln g(x)+c$ P5
"Hence find $\int\ldots$" after a derivative	the result of the previous part	integrate that derivative, then rearrange
Paper 2, f' with no antiderivative ($\sin x^2$, e^{-x^2})	$f(a)$ the GDC	$f(b)=f(a)+\int_a^b f'(x)\,dx$, on the GDC

In the exam 1 *command terms*

Find $\int$: give the constant, $+c$. **Hence**: use the previous part. **Show that**: every step, and the given answer last. **Passes through**, **boundary condition**: a point that fixes c. Check every antiderivative by differentiating it.

VI ON THE GDC: PAPER 2

SL 5.5 · 5.10 · NumWorks · every calculator move of this topic

1 A definite integral

Calculation → Toolbox → Calculus → Integral

Type the function, then the two bounds. The GDC cannot give an antiderivative, but it gives $\int_a^b f'(x)\,dx$: the change of f from a to b.

CALCULATION

```
int(3sin(x^2),x,1,3)
1.389882676
```

2 A value of f from f(a) and f′

Calculation → 2+ ∫ → EXE

$f(b) = f(a) + \int_a^b f'(x)\,\mathrm{d}x$. **On paper:** this line with the numbers in it, then $f(3) = 3.39$.

CALCULATION

```
2+int(3sin(x^2),x,1,3)
3.389882676
```

ANSWERS TO THE APPLY QUESTIONS

I.1 $x^5 + x + c$

I.2 $-\frac{1}{3}x^{-3} + c$

I.3 $\frac{x^3}{3} + x^2 - 3x + c$

I.4 $2\sqrt{x} + c$

I.5 $x - \frac{4}{x} + c$

I.6 $2x^2 + x - 3$

I.7 $2x + \frac{1}{x} + 1$

II.1 $\sin x + 4x + c$

II.2 $2\ln x + c$

II.3 $\frac{1}{5}\sin 5x + c$

II.4 $\ln(4x - 7) + c$

II.5 $-2\cos\frac{x}{2} + c$

II.6 $\frac{1}{2}e^{2x} + \frac{5}{2}$

III.1 $e^{\sin x} + c$

III.2 $\ln\left(x^2 + x + 5\right) + c$

III.3 $\frac{1}{3}e^{x^3} + c$

III.4 $\frac{1}{4}\sin^4 x + c$

III.5 $x\sin x + \cos x + c$

III.6 0.747

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