Antiderivatives

Skills sheet · SL AA 5.5 · 5.10

KEY FACTS

$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$ P1

$\int \mathrm{e}^x = \mathrm{e}^x,\ \int \frac{1}{x} = \ln x,\ \int \sin x = -\cos x,\ \int \cos x = \sin x$ P3

$\int f(ax+b)\,\mathrm{d}x = \frac{1}{a}F(ax+b) + c$ P4

$\int \frac{g'(x)}{g(x)}\,\mathrm{d}x = \ln g(x) + c$ P5

Roots and fractions: powers first W1.
Divide by a, never multiply W2.
No inner derivative, no inspection W3.

WHAT UNLOCKS

- "passes through" → integrate with $+c$, then substitute the point into f.
- a linear inside, $ax+b$ → integrate the outside, divide by a.
- the derivative of the inside in front → by inspection, or $u = g(x)$.
- "Hence find $\int$" after a derivative → integrate the result of the previous part.

WORKED EXAMPLE ①

$f'(x) = 3x^2 + 2$ and $f(1) = 6$. Find $f(x)$. no GDC

$f(x) = x^3 + 2x + c$ — A1A1

$1 + 2 + c = 6$ — M1

$c = 3$: $f(x) = x^3 + 2x + 3$ — A1

Method

How do I use a point?

① Integrate f', with $+c$.
② Put the point into f.
③ Solve for c.
④ Write $f(x)$ in full.

The point goes into f, never into f'.

WORKED EXAMPLE ②

Find $\displaystyle\int \frac{2x}{x^2+1}\,\mathrm{d}x$. no GDC

$2x$ is the derivative of $x^2 + 1$ — (M1)

$\ln(x^2+1) + c$ — A1

Method

How do I inspect?

① Find the inside $g(x)$.
② Is $g'(x)$ in front, up to a constant?
③ Integrate the outside of g.
④ Adjust the constant; check by differentiating.

FLUENCY

●○○ **1.** $\int (4x^3 - 2x)\,\mathrm{d}x$ no GDC

●○○ **2.** $\int x^{-3}\,\mathrm{d}x$ no GDC

●○○ **3.** $\int \mathrm{e}^{4x}\,\mathrm{d}x$ no GDC

●○○ **4.** $\int \sin(3x)\,\mathrm{d}x$ no GDC

●○○ **5.** $\int \frac{1}{2x+5}\,\mathrm{d}x,\ x > -\frac{5}{2}$ no GDC

●○○ **6.** $\int (x+2)(x-2)\,\mathrm{d}x$ no GDC

REASONING

●●● **Question 1.** The gradient of a curve is $f'(x) = 6x - \dfrac{5}{x^2}$, $x > 0$, and $f(1) = 4$. [no GDC]

(a) Find $f(x)$. [4]

●●● **Question 2.** Consider $\displaystyle\int x^2 \left(x^3 + 2\right)^4 \mathrm{d}x$. [no GDC]

(a) Find the integral. [3]

●●● **Question 3.** Let $f(x) = \sin^2 x$. [no GDC]

(a) Find $f'(x)$. [2]

(b) Hence find $\displaystyle\int \sin x \cos x \, \mathrm{d}x$. [2]

PROBLEMS

●●● **Question 4.** A curve has gradient $f'(x) = 2x - 3$. The line $y = x - 1$ is a tangent to the curve. [no GDC]

(a) Find the x-coordinate of the point of contact. [2]

(b) Find $f(x)$. [3]

●●● **Question 5.** A function satisfies $f''(x) = 6x$, $f'(1) = 0$ and $f(1) = 2$. [no GDC]

(a) Find $f'(x)$. [2]

(b) Find $f(x)$. [2]

ANSWERS

1 $x^4 - x^2 + c$

2 $-\frac{1}{2}x^{-2} + c$

3 $\frac{1}{4}e^{4x} + c$

4 $-\frac{1}{3}\cos(3x) + c$

5 $\frac{1}{2}\ln(2x+5) + c$

6 $\frac{x^3}{3} - 4x + c$ (expand first)

Q1 $f(x) = 3x^2 + \frac{5}{x} + c$; $3 + 5 + c = 4$: $f(x) = 3x^2 + \frac{5}{x} - 4$

Q2 $u = x^3 + 2$, $\mathrm{d}u = 3x^2\,\mathrm{d}x$: $\frac{1}{15}\left(x^3+2\right)^5 + c$

Q3 (a) $2\sin x \cos x$ (b) $\frac{1}{2}\sin^2 x + c$

Q4 (a) $2x - 3 = 1$: $x = 2$, the point $(2,\ 1)$ of the line (b) $f(x) = x^2 - 3x + c$, $4 - 6 + c = 1$: $f(x) = x^2 - 3x + 3$

Q5 (a) $f'(x) = 3x^2 + a$, $3 + a = 0$: $f'(x) = 3x^2 - 3$ (b) $f(x) = x^3 - 3x + b$, $1 - 3 + b = 2$: $f(x) = x^3 - 3x + 4$

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