Areas between curves

Skills sheet · SL AA 5.5 · 5.11

KEY FACTS

$f \geq 0$: $A = \int_a^b f(x)\,\mathrm{d}x$ P1

Below the axis: $A = \left|\int_a^b f(x)\,\mathrm{d}x\right|$; split at the roots P2

Two curves: $A = \int_a^b (\text{top} - \text{bottom})\,\mathrm{d}x$ P3

Paper 2: $\int_a^b |f - g|\,\mathrm{d}x$ P4

$\int = 0$ is not "no area" W1.

The top curve can change W2.

Stored limits, not rounded ones W3.

WHAT UNLOCKS

- "enclosed by the curve and the x-axis" $\rightarrow$ the roots are the limits.
- "between the curves" $\rightarrow$ $f(x) = g(x)$ first, then top minus bottom.
- a curve that crosses the axis $\rightarrow$ one integral per piece; add the sizes.
- Paper 2 $\rightarrow$ intersections on the GDC, then $\int |f - g|$.

WORKED EXAMPLE ①

Find the area under $y = x^2 + 1$ from $x = 0$ to $x = 3$. no GDC

$$\int_0^3 \left(x^2 + 1\right)\mathrm{d}x = \left[\frac{x^3}{3} + x\right]_0^3 \quad \text{M1A1}$$

$$= 9 + 3 = 12 \quad \text{A1}$$

Method

How do I find an area?

① Sketch: where is the region?
② The limits: roots or intersections.
③ Top minus bottom, or $|f|$.
④ One integral per piece; add.

An area is never negative.

WORKED EXAMPLE ②

Find the area enclosed by $y = x^2$ and $y = x$. no GDC

$x^2 = x$: $x = 0, x = 1$ (M1)A1

$$\int_0^1 \left(x - x^2\right)\mathrm{d}x = \frac{1}{2} - \frac{1}{3} \quad \text{M1A1}$$

$$= \frac{1}{6} \quad \text{A1}$$

Method

Which one is on top?

① Pick an x between the limits.
② Compare $f(x)$ and $g(x)$ there.
③ The larger is the top.
④ Or use $|f - g|$ on the GDC.

FLUENCY

●○○ **1.** The area under $y = 3x^2$, $1 \leq x \leq 2$ no GDC

●○○ **2.** The area under $y = \frac{1}{x}$, $1 \leq x \leq \mathrm{e}$ no GDC

●○○ **3.** The area between $y = x^2 - 4$ and the x-axis no GDC

●○○ **4.** The area under $y = \sin x$, $0 \leq x \leq \pi$ no GDC

●○○ **5.** The area between $y = x$ and $y = x^3$, $0 \leq x \leq 1$ no GDC

●○○ **6.** Where do $y = x^2$ and $y = x + 2$ meet? no GDC

REASONING

●●● **Question 1.** The graphs of $y = x^2$ and $y = x + 2$ enclose a region. [no GDC]

(a) Find its area. [4]

●●● **Question 2.** Let $f(x) = x^3 - x^2 - 2x$. [no GDC]

(a) Factorise $f(x)$ and write down the x-intercepts. [2]

(b) Find the total area enclosed by the graph and the x-axis. [5]

●●● **Question 3.** The region R is bounded by $y = e^x$, the x-axis, $x = 0$ and $x = \ln 3$. [no GDC]

(a) Find the area of R. [3]

PROBLEMS

●●● **Question 4.** The area under $y = kx^2$ from $x = 0$ to $x = 3$ is 18. [no GDC]

(a) Find k. [3]

●●● **Question 5.** The region under $y = x$ for $0 \leq x \leq 4$ is cut into two parts of equal area by the line $x = a$. [no GDC]

(a) Find the exact value of a. [4]

ANSWERS

1 $\left[x^3\right]_1^2 = 7$

2 1

3 $\left|\int_{-2}^{2}\left(x^2-4\right)\mathrm{d}x\right| = \frac{32}{3}$

4 2

5 $\int_0^1\left(x-x^3\right)\mathrm{d}x = \frac{1}{4}$

6 $x=-1$ and $x=2$

Q1 $\int_{-1}^{2}\left(x+2-x^2\right)\mathrm{d}x = \frac{10}{3}+\frac{7}{6} = \frac{9}{2}$

Q2 (a) $x(x-2)(x+1)$: $-1, 0, 2$ (b) $\int_{-1}^{0} f = \frac{5}{12}$, $\int_0^2 f = -\frac{8}{3}$: area $\frac{5}{12}+\frac{8}{3} = \frac{37}{12}$

Q3 $\left[\mathrm{e}^x\right]_0^{\ln 3} = 3-1 = 2$

Q4 $\left[\frac{kx^3}{3}\right]_0^3 = 9k = 18$: $k=2$

Q5 the whole area is 8; $\frac{a^2}{2} = 4$: $a = 2\sqrt{2}$

Written and verified by a practising IB mathematics teacher, produced with AI assistance; every result recomputed by software, every figure computed from its function. Not affiliated with or endorsed by the International Baccalaureate Organization. © 2026 Hence