Definite integrals

A definite integral is a number: the change of an antiderivative between two values, $F(b) - F(a)$. For a positive function it is also an area, the limit of thinner and thinner rectangles. The exam asks for exact values on Paper 1, for values to three significant figures on Paper 2, and uses the properties of integrals to hide an unknown.

I THE FUNDAMENTAL THEOREM

SL 5.5 · 5.11 · $\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$

I.1 $F(b) - F(a)$

Definition 1 *definite integral*

F an antiderivative of f: $\displaystyle\int_a^b f(x)\,\mathrm{d}x = \Big[F(x)\Big]_a^b = F(b) - F(a)$, a number. No $+c$: it would cancel.

Watch out 1

$F(b) - F(a)$ in full, each value in its own brackets: top limit first, then minus the whole of the bottom one.

Example 1

Find $\displaystyle\int_1^3 (3x^2 + 2x)\,\mathrm{d}x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Antiderivative in square brackets	$\Big[x^3 + x^2\Big]_1^3$
Step 2 Top minus bottom, each in brackets	$(27 + 9) - (1 + 1)$
Step 3 Compute	34

APPLY

1 $\displaystyle\int_0^2 (4x - 1)\,\mathrm{d}x$

2 $\displaystyle\int_1^4 \frac{1}{x^2}\,\mathrm{d}x$

3 $\displaystyle\int_{-1}^1 x^3\,\mathrm{d}x$

I.2 Exact values with e, ln, sin, cos

Example 2

Find the exact value of $\int_0^{\frac{\pi}{3}} \sin x \, dx$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Antiderivative	$\left[-\cos x\right]_0^{\frac{\pi}{3}}$
Step 2 Top minus bottom	$-\cos\frac{\pi}{3} - (-\cos 0) = -\frac{1}{2} + 1$
Step 3 Exact value	$\frac{1}{2}$

APPLY

4 $\int_0^2 e^{\frac{x}{2}} \, dx$

5 $\int_1^{e^2} \frac{1}{x} \, dx$

6 $\int_0^{\pi} \cos\frac{x}{2} \, dx$

I.3 An area under a curve

Property 1 *an area*

When $f(x) \geq 0$ on $[a, b]$, $\int_a^b f(x) \, dx$ is the area between the graph, the x-axis and $x = a$, $x = b$: the limit of sums of thinner rectangles.

Example 3

Find the area of the region under $y = x^2 + 1$, above the x-axis, from $x = 0$ to $x = 3$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $f \geq 0$: the area is the integral	$\int_0^3 (x^2 + 1) \, dx$
Step 2 Antiderivative	$\left[\frac{x^3}{3} + x\right]_0^3$
Step 3 Compute	$9 + 3 = 12$

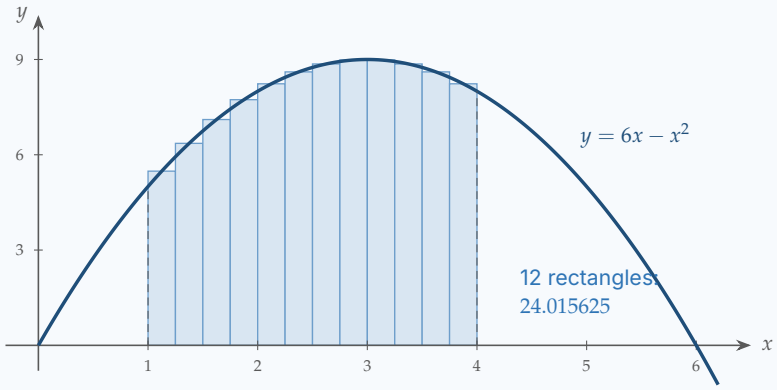


Figure 1 — $f(x) = 6x - x^2$ on $1 \leq x \leq 4$: rectangles of width 0.25 add up to 24.015625; thinner ones tend to the integral, 24.

APPLY

7 The area under $y = \sqrt{x}$ from 0 to 9.

8 The area under $y = e^x$ from 0 to 1.

II PROPERTIES, AND AN UNKNOWN

SL 5.11 · sums, multiples, limits reversed or joined

II.1 The properties

Property 2 *properties*

$$\int_a^b (kf+g) = k\int_a^b f + \int_a^b g,\ \int_a^b k\,\mathrm{d}x = k(b-a),\ \int_b^a f = -\int_a^b f,\ \int_a^b f + \int_b^c f = \int_a^c f.$$

Watch out 2

A constant is integrated too: $\int_1^4 (f(x)+3)\,\mathrm{d}x = \int_1^4 f + 3\times 3$. Reversing the limits changes the sign of the whole integral.

Example 4

$\int_0^3 f(x)\,\mathrm{d}x = 5$. Find $\int_0^3 (2f(x)-1)\,\mathrm{d}x$ and $\int_3^0 f(x)\,\mathrm{d}x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Split, factor out	$2\times 5 - \int_0^3 1\,\mathrm{d}x$
Step 2 The constant: $k(b-a)$	$10-3=7$
Step 3 Limits reversed: change the sign	-5

APPLY

1 $\int_0^2 f = 4$, $\int_2^5 f = 1$: find $\int_0^5 f$.

2 Same: find $\int_2^5 (3f(x)+2)\,\mathrm{d}x$.

II.2 An unknown limit

Example 5

Find $k>0$ such that $\int_0^k (3x^2+1)\,\mathrm{d}x = 10$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Integrate, with k as the top limit	$\left[x^3+x\right]_0^k = k^3+k$
Step 2 Write the equation	$k^3+k=10$
Step 3 Solve	$k=2$

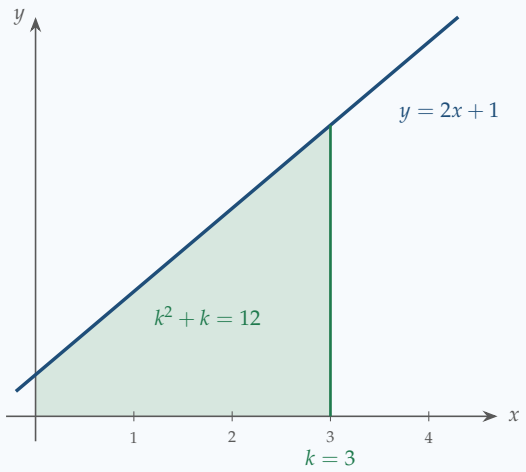


Figure 2 — $\int_0^k (2x+1)\,\mathrm{d}x = k^2 + k$: the area grows with k and equals 12 at $k = 3$.

APPLY

3 $\displaystyle\int_1^k 2x\,\mathrm{d}x = 15, k > 1.$

4 $\displaystyle\int_0^k \mathrm{e}^x\,\mathrm{d}x = 4.$

III EXACT VALUES WITH AN INSIDE

SL 5.10 · 5.11 · substitution and its limits

III.1 By inspection, then the limits

Property 3 *the limits follow the variable*

With $u = g(x)$ the limits change too: $\displaystyle\int_a^b f\big(g(x)\big)g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u$. Or the antiderivative in x by inspection, limits in x.

Example 6

Find the exact value of $\displaystyle\int_0^1 2x\,\mathrm{e}^{x^2}\,\mathrm{d}x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 By inspection: $2x$ is the inner derivative	$\Big[\mathrm{e}^{x^2}\Big]_0^1$
Step 2 Top minus bottom	$\mathrm{e} - 1$

APPLY

1 $\displaystyle\int_0^2 \frac{2x}{x^2+4}\,\mathrm{d}x$

2 $\displaystyle\int_0^{\frac{\pi}{2}} \cos x \sin x\,\mathrm{d}x$

III.2 Change the limits with u

Watch out 3

Never an integral in u with limits in x. On Paper 1 an exact value stays exact: $\ln\frac{5}{2}$, $\sqrt{3}$, $\frac{15}{8}$.

Example 7

Use $u = x^3 + 1$ to find the exact value of $\displaystyle\int_0^1 x^2\left(x^3+1\right)^2\,\mathrm{d}x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $\mathrm{d}u$	$\mathrm{d}u = 3x^2\,\mathrm{d}x$
Step 2 New limits	$x = 0 \to u = 1,\ x = 1 \to u = 2$
Step 3 Integral in u	$\displaystyle\frac{1}{3}\int_1^2 u^2\,\mathrm{d}u = \frac{1}{3}\Big[\frac{u^3}{3}\Big]_1^2$
Step 4 Compute	$\displaystyle\frac{7}{9}$

APPLY

3 Use $u = x^2 - 1$: $\displaystyle\int_1^2 2x\left(x^2-1\right)^3 \mathrm{d}x$.

III.3 On the GDC

Example 8

Find $\displaystyle\int_0^2 \sqrt{x^3+1}\,\mathrm{d}x$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the integral	$\displaystyle\int_0^2 \sqrt{x^3+1}\,\mathrm{d}x$
Step 2 Integral on the GDC	3.24131 . . .
Step 3 Answer, 3 s.f.	3.24

APPLY

4 $\displaystyle\int_{-1}^1 \mathrm{e}^{-x^2}\,\mathrm{d}x$

IV EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A F(b) – F(a)

EXAM-STYLE QUESTION 1

Paper 1 · no calculator · [Maximum mark: 4]

Find the value of $\int_1^2 (6x^2 - 2x)\,\mathrm{d}x$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Antiderivative in square brackets	$\left[2x^3 - x^2\right]_1^2$	A1A1
	A1 for each term.	
Step 2 Top minus bottom	$(16 - 4) - (2 - 1)$	M1
	Both limits, the top one first.	
Step 3 Compute	$= 11$	A1

EXAM-STYLE QUESTION 2

Paper 1 · no calculator · [Maximum mark: 6]

Find the exact value of each integral.

(a) $\int_0^{\ln 3} \mathrm{e}^{2x}\,\mathrm{d}x$ [3]

(b) $\int_0^{\frac{\pi}{6}} 4\cos(2x)\,\mathrm{d}x$ [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Divide by 2	$\left[\frac{1}{2}\mathrm{e}^{2x}\right]_0^{\ln 3}$	A1
Step 2 Top minus bottom: $\mathrm{e}^{2\ln 3} = 9$	$\frac{1}{2}\mathrm{e}^{2\ln 3} - \frac{1}{2}\mathrm{e}^0 = \frac{9}{2} - \frac{1}{2}$	M1
	$\mathrm{e}^{2\ln 3} = \left(\mathrm{e}^{\ln 3}\right)^2 = 9$.	
Step 3 Compute	$= 4$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Divide by 2	$\left[2\sin(2x)\right]_0^{\frac{\pi}{6}}$	A1
Step 2 Top minus bottom, exact values	$2\sin\frac{\pi}{3} - 2\sin 0 = 2 \times \frac{\sqrt{3}}{2}$	M1
Step 3 Simplify	$= \sqrt{3}$	A1

B Properties and an unknown limit

EXAM-STYLE QUESTION 3 Paper 1 · no calculator · [Maximum mark: 6]

The functions f and g are such that $\int_1^4 f(x)\,\mathrm{d}x = 7$ and $\int_1^4 g(x)\,\mathrm{d}x = -2$.

(a) Find $\int_1^4 \left(2f(x)+3\right)\mathrm{d}x$. [3]

(b) Find $\int_4^1 \left(f(x)-g(x)\right)\mathrm{d}x$. [2]

It is also given that $\int_1^6 f(x)\,\mathrm{d}x = 10$.

(c) Find $\int_4^6 f(x)\,\mathrm{d}x$. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Split, the constant $\times(4-1)$	$2\int_1^4 f(x)\,\mathrm{d}x + \int_1^4 3\,\mathrm{d}x$ *Split the integral.*	M1
Step 2 Substitute	$2\times 7 + 3\times 3$ $\int_1^4 3\,\mathrm{d}x = 3(4-1) = 9$.	A1
Step 3 Compute	$= 23$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Limits reversed: change the sign	$-\int_1^4 \left(f(x)-g(x)\right)\mathrm{d}x = -\left(7-(-2)\right)$ *Reversed limits: change the sign.*	M1
Step 2 Compute	$= -9$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Join the limits	$10 - 7 = 3$ $\int_1^4 + \int_4^6 = \int_1^6$.	A1

EXAM-STYLE QUESTION 4 Paper 1 · no calculator · [Maximum mark: 4]

It is given that $\int_0^k (2x+1)\,\mathrm{d}x = 12$, where $k > 0$. Find the value of k.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Integrate with k	$\left[x^2 + x\right]_0^k = k^2 + k$	A1
Step 2 Write the equation	$k^2 + k = 12$ *Set the integral equal to the value.*	M1
Step 3 Factorise	$(k+4)(k-3) = 0$	(A1)
Step 4 Keep $k > 0$	$k = 3$ *$k = -4$ is rejected: $k > 0$.*	A1

C Exact values with an inside

EXAM-STYLE QUESTION 5 Paper 1 · no calculator · [Maximum mark: 4]

Find $\int_1^2 \dfrac{2x}{x^2+1}\,\mathrm{d}x$, giving your answer in the form $\ln k$, where $k \in \mathbb{Q}$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Top: the derivative of the bottom	the numerator is the derivative of x^2+1	(M1)
Step 2 A logarithm	$\left[\ln\left(x^2+1\right)\right]_1^2$	A1
Step 3 Top minus bottom	$\ln 5 - \ln 2$	A1
Step 4 One logarithm	$= \ln \dfrac{5}{2}$ *Accept* $\ln 2.5$.	A1

EXAM-STYLE QUESTION 6 Paper 1 · no calculator · [Maximum mark: 4]

Use the substitution $u = x^2 + 1$ to find the exact value of $\int_0^1 x\left(x^2+1\right)^3\,\mathrm{d}x$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $\mathrm{d}u$	$\mathrm{d}u = 2x\,\mathrm{d}x$	M1
Step 2 New limits	$x = 0 \to u = 1,\ x = 1 \to u = 2$ *The limits in u.*	A1
Step 3 Integral in u	$\dfrac{1}{2}\int_1^2 u^3\,\mathrm{d}u = \dfrac{1}{8}\left[u^4\right]_1^2$	A1
Step 4 Compute	$= \dfrac{15}{8}$	A1

EXAM-STYLE QUESTION 7 — Paper 1 · no calculator · [Maximum mark: 3]

Show that $\int_0^{\frac{\pi}{2}} \sin x \cos^2 x \, dx = \frac{1}{3}$.

WORKING

WHAT YOU DO	WHAT YOU WRITE	
Step 1 By inspection	by inspection: the derivative of $\cos^3 x$ is $-3 \sin x \cos^2 x$	M1
Step 2 Antiderivative	$\left[-\frac{1}{3}\cos^3 x\right]_0^{\frac{\pi}{2}}$	A1
Step 3 Top minus bottom	$= 0 - \left(-\frac{1}{3}\right)$ *$\cos\frac{\pi}{2} = 0$, $\cos 0 = 1$.*	A1
Step 4 Reach the given value	$= \frac{1}{3}$	AG

D With the GDC (Paper 2)

EXAM-STYLE QUESTION 8 — Paper 2 · calculator · [Maximum mark: 6]

Let $f(x) = \ln(x^2 + 1)$.

(a) Find $\int_0^3 f(x) \, dx$. [2]

(b) Hence find $\int_0^3 (f(x) + 2) \, dx$. [2]

(c) Hence find $\int_3^0 2f(x) \, dx$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the integral	$\int_0^3 \ln(x^2 + 1) \, dx$	(M1)
Step 2 On the GDC	$= 3.41 \quad (3.40584\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Add 2×3	$3.40584\ldots + 2 \times 3$ *$\int_0^3 2 \, dx = 6$, not 2.*	M1
Step 2 Compute	$= 9.41$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Reverse, multiply by 2	$-2 \times 3.40584\ldots$ *Reversed limits and a factor 2.*	M1
Step 2 Compute	$= -6.81$	A1

EXAM-STYLE QUESTION 9

Paper 2 · calculator · [Maximum mark: 5]

It is given that $\int_0^k e^{0.5x}\,dx = 10$, where $k > 0$.

(a) Show that $2e^{0.5k} - 2 = 10$. [3]

(b) Find the value of k. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Antiderivative	$\left[2e^{0.5x}\right]_0^k$ *Divide by 0.5: multiply by 2.*	A1
Step 2 Top minus bottom	$2e^{0.5k} - 2e^0$ *Both limits.*	M1
Step 3 $e^0 = 1$	$2e^0 = 2$ *The value at 0 is 2, not 0.*	A1
Step 4 Reach the given form	$2e^{0.5k} - 2 = 10$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Isolate the exponential	$e^{0.5k} = 6$	(M1)
Step 2 Take ln	$k = 2\ln 6 = 3.58\ \ (3.58351\ldots)$	A1

V WHICH METHOD? READ THE QUESTION

5.11 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"find $\int_a^b$", Paper 1	an antiderivative F	$[F(x)]_a^b = F(b) - F(a)$, each value in brackets; no $+c$ D1
"the area under the curve", $f \geq 0$	a positive integral	the definite integral between the two limits P1
$\int f$ is given; find $\int (kf + c)$	the properties	$k\times$ the given value $+ c(b-a)$ P2
the limits in the other order	$\int_b^a = -\int_a^b$	change the sign of the whole integral P2
"find k" with $\int_0^k \ldots = a$ value	an equation in k	integrate, put in k, solve; reject the values the question excludes
an inside and its derivative, exact value	$u = g(x)$	new limits in u, or back to x first P3
"Show that $\int \ldots =$"	the value to reach	every step written; the given value as the last line
Paper 2, no antiderivative	the GDC	the integral written out, then its value to 3 s.f.

In the exam 1 *command terms*

Find the exact value: no decimals, keep ln, $\sqrt{}$, π, fractions. **Show that**: every step, the given value last. **Hence**: use the previous part. **Write down**: no working needed. Paper 2: three significant figures.

VI ON THE GDC: PAPER 2

SL 5.5 · 5.11 · NumWorks · every calculator move of this topic

1 A definite integral

Calculation → Toolbox → Calculus → Integral

Type the function, then the lower and the upper limit. **On paper:** the integral written in full, then 0.882.

CALCULATION

```
int(e^(-x^2),x,0,2)
0.8820813908
```

2 Properties with a value from the GDC

Calculation → Ans → ×2 + 6

Keep the full value in Ans and apply the properties to it: **On paper:** $2\int_0^3 f + \int_0^3 2\,\mathrm{d}x = 12.8$.

CALCULATION

```
int(ln(x^2+1),x,0,3)
3.405846824
2*Ans+6
12.81169365
```

ANSWERS TO THE APPLY QUESTIONS

I.1 6

I.2 $\frac{3}{4}$

I.3 0

I.4 $2\mathrm{e} - 2$

I.5 2

I.6 2

I.7 18

I.8 $\mathrm{e} - 1$

II.1 5

II.2 9

II.3 $k = 4$

II.4 $k = \ln 5$

III.1 $\ln 2$

III.2 $\frac{1}{2}$

III.3 $\frac{81}{4}$

III.4 1.49

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