Marked example 3

Paper 1 style · No calculator

3 6 marks SL 5.10 · 5.11 ●●○ Typical

Find the exact value of each integral.

(a) $\int_1^{\mathrm{e}} \frac{\ln x}{x}\,\mathrm{d}x$, using the substitution $u = \ln x$. **[4]**

(b) $\int_0^1 \mathrm{e}^{3x}\,\mathrm{d}x$ **[2]**

MARKSCHEME (a)

$\mathrm{d}u = \frac{1}{x}\,\mathrm{d}x$ **M1**

$x = 1 \to u = 0,\ x = \mathrm{e} \to u = 1$ **A1**

$\int_0^1 u\,\mathrm{d}u = \left[\frac{u^2}{2}\right]_0^1$ **A1**

$= \frac{1}{2}$ **A1**

Total 4 marks

Note The limits change with the variable: $\ln 1 = 0$, $\ln \mathrm{e} = 1$.

MARKSCHEME (b)

$\left[\frac{1}{3}\mathrm{e}^{3x}\right]_0^1$ **A1**

$= \frac{\mathrm{e}^3 - 1}{3}$ **A1**

Total 2 marks

Note The value at 0 is $\frac{1}{3}$, not 0.

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