Definite integrals

Skills sheet · SL AA 5.5 · 5.11

KEY FACTS

$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$, no $+c$ D1
$f \geq 0$: the area under the curve P1
$\int_b^a f = -\int_a^b f$, $\int_a^b + \int_b^c = \int_a^c$, $\int_a^b k = k(b-a)$ P2
$u = g(x)$: the limits become $g(a)$, $g(b)$ P3

Each value in its own brackets W1.
A constant integrates to $k(b-a)$ W2.
Limits in u *for an integral in* u W3.

WHAT UNLOCKS

- "exact value" → an antiderivative, then $F(b) - F(a)$; keep ln, $\sqrt{}$, e, π.
- a given integral → the properties: split, factor out, reverse, join.
- "find k" → integrate, put in k, solve; reject what the question excludes.
- Paper 2 → the integral written in full, then the GDC value to 3 s.f.

WORKED EXAMPLE ①

Find $\int_1^2 (4x^3 - 1)\,\mathrm{d}x$. no GDC

$\left[x^4 - x\right]_1^2$ A1
$(16 - 2) - (1 - 1)$ M1
$= 14$ A1

Method
How do I evaluate?
① Antiderivative, no $+c$.
② Square brackets with the limits.
③ $F(\text{top}) - F(\text{bottom})$, brackets.
④ Simplify; exact on Paper 1.
$F(0)$ ***is not always*** 0.

WORKED EXAMPLE ②

Find $\int_0^1 6x\left(x^2+1\right)^2\,\mathrm{d}x$ **with** $u = x^2 + 1$. no GDC

$\mathrm{d}u = 2x\,\mathrm{d}x$; u from 1 to 2 M1A1
$3\int_1^2 u^2\,\mathrm{d}u = \left[u^3\right]_1^2$ A1
$= 7$ A1

Method
How do I change the limits?
① $u = g(x)$ and $\mathrm{d}u$.
② u at each limit in x.
③ Integrate in u between them.
④ No return to x needed.

FLUENCY

●○○ **1.** $\int_0^3 x^2\,\mathrm{d}x$ no GDC
●○○ **2.** $\int_1^4 \sqrt{x}\,\mathrm{d}x$ no GDC
●○○ **3.** $\int_0^\pi \sin x\,\mathrm{d}x$ no GDC
●○○ **4.** $\int_0^1 \mathrm{e}^{2x}\,\mathrm{d}x$, exact no GDC
●○○ **5.** $\int_1^\mathrm{e} \frac{1}{x}\,\mathrm{d}x$ no GDC
●○○ **6.** $\int_2^2 \left(x^5 + 7\right)\,\mathrm{d}x$ no GDC

REASONING

●●● **Question 1.** It is given that $\int_1^3 f(x)\,\mathrm{d}x = 4$. [no GDC]

(a) Find $\int_1^3 \left(5 - 2f(x)\right)\,\mathrm{d}x$. [3]

(b) Write down $\int_3^1 f(x)\,\mathrm{d}x$. [1]

●●● **Question 2.** It is given that $\int_0^a \left(3x^2 + 2\right)\,\mathrm{d}x = 12$, where $a > 0$. [no GDC]

(a) Find the value of a. [4]

●●● **Question 3.** Consider $\int_0^{\frac{\pi}{4}} \cos(2x)\,\mathrm{d}x$. [no GDC]

(a) Show that its value is $\frac{1}{2}$. [3]

PROBLEMS

●●● **Question 4.** Consider $\int_0^{\ln 2} \left(\mathrm{e}^x - \mathrm{e}^{-x}\right)\,\mathrm{d}x$. [no GDC]

(a) Find its exact value. [4]

●●● **Question 5.** It is given that $\int_1^p \frac{1}{x}\,\mathrm{d}x = 2$, where $p > 1$. [no GDC]

(a) Find the exact value of p. [3]

ANSWERS

1 $\left[\frac{x^3}{3}\right]_0^3 = 9$

2 $\left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$

3 $\left[-\cos x\right]_0^{\pi} = 1 + 1 = 2$

4 $\frac{e^2-1}{2}$

5 $\ln e - \ln 1 = 1$

6 0: equal limits

Q1 (a) $5 \times 2 - 2 \times 4 = 2$ (b) -4

Q2 $a^3 + 2a = 12$, $(a-2)(a^2+2a+6) = 0$: $a = 2$

Q3 $\left[\frac{1}{2}\sin(2x)\right]_0^{\pi/4} = \frac{1}{2}\sin\frac{\pi}{2} - 0 = \frac{1}{2}$

Q4 $\left[e^x + e^{-x}\right]_0^{\ln 2} = \left(2 + \frac{1}{2}\right) - (1+1) = \frac{1}{2}$

Q5 $\ln p - \ln 1 = 2$: $p = e^2$

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