Kinematics with integrals

Skills sheet · SL AA 5.9

KEY FACTS

$v = \int a\,\mathrm{d}t$, $s = \int v\,\mathrm{d}t$; constants from $v(0)$, $s(0)$ P1

Displacement: $\int_{t_1}^{t_2} v\,\mathrm{d}t$ P2

Distance: $\int_{t_1}^{t_2} |v|\,\mathrm{d}t$; Paper 1: split at $v = 0$ P3

Back at the start: $\int_0^T v\,\mathrm{d}t = 0$ P4

Two constants, two conditions W1.

Displacement is not distance W2.

$|v|$ for a distance, v for a displacement W3.

WHAT UNLOCKS

- "from rest" $\rightarrow$ $v(0) = 0$; "from O" $\rightarrow$ $s(0) = 0$.
- "displacement between two times" $\rightarrow$ $\int v$, with its sign.
- "distance travelled" $\rightarrow$ the zeros of v first; add the sizes.
- "returns to its starting point" $\rightarrow$ $s(T) = s(0)$, solved for $T > 0$.

WORKED EXAMPLE ①

$v(t) = 3t^2 - 2t$ **and** $s(0) = 4$**. Find** $s(t)$**.** no GDC

$s(t) = t^3 - t^2 + c$ A1A1

$s(0) = 4$: $c = 4$ M1

$s(t) = t^3 - t^2 + 4$ A1

Method

How do I go back to s?

① Integrate a, with $+c$.
② $v(0)$ gives c.
③ Integrate v, with $+d$.
④ $s(0)$ gives d.

"From rest" is $v(0) = 0$***, not*** $s(0) = 0$**.**

WORKED EXAMPLE ②

$v(t) = 2t - 4$, $0 \leq t \leq 3$**. Find the distance travelled.** no GDC

$v = 0$ at $t = 2$ (M1)

$\int_0^2 v\,\mathrm{d}t = -4$, $\int_2^3 v\,\mathrm{d}t = 1$ A1

$4 + 1 = 5$ m A1

Method

How do I find a distance?

① The zeros of v in the interval.
② $\int v$ on each piece.
③ Add the sizes.
④ Paper 2: $\int |v|$ directly.

FLUENCY

●○○ **1.** $a = 6$, $v(0) = 2$: find $v(t)$. no GDC

●○○ **2.** $v = 3t^2$, $s(0) = 0$: find $s(t)$. no GDC

●○○ **3.** $v = \mathrm{e}^t$, $s(0) = 1$: find $s(t)$. no GDC

●○○ **4.** $v = 2t$: the displacement from $t = 0$ to $t = 3$. no GDC

●○○ **5.** $v = \cos t$, starts at O: find $s(t)$. no GDC

●○○ **6.** What does "starts from rest" give? no GDC

REASONING

●●● **Question 1.** A particle starts from rest at O with acceleration $a(t) = 6t - 6$. [no GDC]

(a) Find $v(t)$ and $s(t)$. [4]

(b) Find when it is next at rest, and its displacement then. [2]

●●● **Question 2.** A particle starts at O with velocity $v(t) = 2\sin\left(\frac{t}{2}\right)$. [no GDC]

(a) Find $s(t)$. [3]

(b) Find $s(\pi)$. [1]

●●● **Question 3.** A particle has velocity $v(t) = t - 2$, for $0 \leq t \leq 4$. [no GDC]

(a) Find the displacement and the distance travelled. [4]

PROBLEMS

●●● **Question 4.** A particle has velocity $v(t) = kt(4 - t)$, for $0 \leq t \leq 4$, and its displacement over these 4 seconds is 16 m. [no GDC]

(a) Find k. [3]

●●● **Question 5.** Two particles start together at O: $v_A(t) = 2t$ and $v_B(t) = 6 - t$. [no GDC]

(a) Find when they meet again. [4]

ANSWERS

1 $v = 6t + 2$

2 $s = t^3$

3 $s = \mathrm{e}^t \;\; (c = 0)$

4 $\left[t^2\right]_0^3 = 9$ m

5 $s = \sin t$

6 $v(0) = 0$

Q1 (a) $v = 3t^2 - 6t,\; s = t^3 - 3t^2$ (b) $t = 2,\; s(2) = -4$ m

Q2 (a) $s = -4\cos\left(\frac{t}{2}\right) + 4$ (b) 4 m

Q3 displacement $\left[\frac{t^2}{2} - 2t\right]_0^4 = 0$; distance $2 + 2 = 4$ m

Q4 $k\left[2t^2 - \frac{t^3}{3}\right]_0^4 = \frac{32k}{3} = 16$: $k = 1.5$

Q5 $s_A = t^2, s_B = 6t - \frac{t^2}{2}$; $\frac{3}{2}t^2 = 6t$: $t = 4$

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