Arithmetic sequences and series

An arithmetic sequence adds the same number at every step: its terms lie on a straight line. Two numbers describe it completely, the first term and the common difference, and two formulas answer every question: the nth term and the sum of the first n terms.

I ARITHMETIC SEQUENCES

SL 1.2 · $u_n = u_1 + (n-1)d$, the common difference

I.1 The common difference and u_n

Definition 1 *arithmetic sequence*

A sequence is **arithmetic** when $u_{n+1} - u_n = d$ is constant, the **common difference**. Its nth term is $u_n = u_1 + (n-1)d$.

Example 1

Consider the sequence $7,\ 11,\ 15,\ \ldots$ Find the common difference, a formula for u_n, and u_{20}.

WHAT YOU DO	WHAT YOU WRITE
Step 1 d: subtract consecutive terms	$d = 11 - 7 = 4$
Step 2 Write $u_1 + (n-1)d$	$u_n = 7 + 4(n-1) = 4n + 3$
Step 3 Substitute $n = 20$	$u_{20} = 7 + 19 \times 4 = 83$

19 steps, not 20.

APPLY

1 $30,\ 26,\ 22,\ \ldots$: find d and u_{10}.

2 $2.5,\ 4,\ 5.5,\ \ldots$: find u_8.

3 $u_n = 5n - 2$: first term and common difference.

I.2 Is x a term?

Watch out 1

The nth term has $n-1$ steps: $u_{20} = u_1 + 19d$. And n counts terms: an answer like $n = 12.5$ means no term has that value.

Example 2

Is 200 a term of $7,\ 11,\ 15,\ \ldots$? Is 199?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write $u_n = x$	$4n + 3 = 200$
Step 2 Solve for n	$n = 49.25$
Step 3 Is n a positive integer?	no: 200 is not a term
Step 4 Same for 199	$4n + 3 = 199$: $n = 49$, so $199 = u_{49}$

APPLY

4 Is 100 a term of $4,\ 10,\ 16,\ \ldots$?

5 Is -51 a term of $61,\ 57,\ 53,\ \ldots$?

I.3 From two terms

Property 1 *two terms are enough*

Between u_k and u_m there are $m-k$ steps of d: $u_m - u_k = (m-k)d$. Then $u_1 = u_k - (k-1)d$.

Example 3

In an arithmetic sequence, $u_4 = 19$ and $u_{10} = 7$. Find d and u_1.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Count the steps between the terms	$10 - 4 = 6$ steps: $6d = 7 - 19 = -12$
Step 2 Solve for d	$d = -2$
Step 3 Go back to u_1: 3 steps	$u_1 = 19 - 3 \times (-2) = 25$

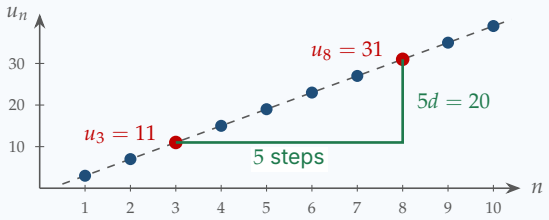


Figure 1 — Exam-style question 1 on the graph: $u_n = 3 + 4(n-1)$, points on a line of gradient $d = 4$; $u_3 = 11$ and $u_8 = 31$ are five steps apart.

APPLY

6 $u_2 = 9$, $u_7 = 34$: find d and u_1.

7 $u_5 = 20$, $u_{12} = 48$: find d and u_1.

II ARITHMETIC SERIES

SL 1.2 · S_n, sigma notation, applications

II.1 The sum S_n

Definition 2 *arithmetic series*

The sum of the first n terms: $S_n = \frac{n}{2}\big(2u_1 + (n-1)d\big) = \frac{n}{2}(u_1 + u_n)$. In **sigma notation**, $\sum_{k=1}^{n} u_k = u_1 + u_2 + \cdots + u_n$.

Example 4

Find the sum of the first 20 terms of $7,\ 11,\ 15,\ \ldots$

WHAT YOU DO	WHAT YOU WRITE
Step 1 Read u_1, d, n	$u_1 = 7, d = 4, n = 20$
Step 2 Substitute in S_n	$S_{20} = \frac{20}{2}\big(14 + 19 \times 4\big)$
Step 3 Compute	$= 10 \times 90 = 900$

APPLY

1 $3,\ 8,\ 13,\ \ldots$: find S_{15}.

2 $50,\ 47,\ 44,\ \ldots$: find S_{12}.

II.2 The sum from the first and last terms

Example 5

Find $6 + 10 + 14 + \cdots + 102$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 How many terms? Solve $u_n = 102$	$6 + 4(n-1) = 102$: $n = 25$
Step 2 Use $\frac{n}{2}(u_1 + u_n)$	$\frac{25}{2}(6 + 102)$
Step 3 Compute	$= 1350$

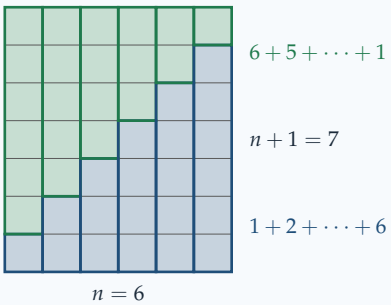


Figure 2 — Why $S_n = \frac{n}{2}(u_1 + u_n)$: the sum and the same sum turned round fit together, each column a first term plus a last term.

APPLY

3 $2+4+6+\cdots+100$

4 $10+13+16+\cdots+70$

II.3 Sigma notation

Property 2 *reading a sigma*

$\sum_{k=1}^{n}(ak+b)$: put $k=1$ for the first term; the common difference is a; there are n terms, the last one at $k=n$.

Example 6

Find $\sum_{k=1}^{12}(5k-2)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 First and last terms: $k=1$, $k=12$	$u_1=3,\ u_{12}=58$
Step 2 Number of terms	12
Step 3 Use $\frac{n}{2}(u_1+u_n)$	$\frac{12}{2}(3+58)=366$

APPLY

5 $\sum_{k=1}^{10}(2k+7)$

6 $\sum_{k=1}^{30}(100-3k)$

II.4 One term, or the sum?

Watch out 2

S_n is a sum, u_n is one term: "seats in the 15th row" is u_{15}; "seats in 15 rows" is S_{15}.

Example 7

A hall has 12 seats in the first row, and each row has 2 more seats than the row in front. Find the number of seats in the 10th row, and in the first 10 rows.

WHAT YOU DO	WHAT YOU WRITE
Step 1 One row: u_{10}	$12+9\times 2=30$
Step 2 All the rows: S_{10}	$\frac{10}{2}(12+30)$
Step 3 Compute	$=210$ seats

APPLY

7 Week 1: €5 saved, then €3 more each week. Saved in week 12?

8 Same savings: total after 12 weeks?

II.5 Finding n from a sum

Example 8

Find the least n such that the sum of the first n terms of $4,\ 7,\ 10,\ \ldots$ is more than 500.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write S_n with n	$S_n = \frac{n}{2}\big(8 + 3(n-1)\big) > 500$
Step 2 Solve on the GDC	$n > 17.4 \ \ (17.4119\ldots)$
Step 3 Next integer up, and check	$S_{17} = 476$, $S_{18} = 531$: $\ n = 18$

APPLY

9 $1 + 5 + 9 + \cdots$: least n with $S_n > 1000$.

10 $20 + 18 + 16 + \cdots$: how many terms give a sum of 110?

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Arithmetic sequences

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 5]

In an arithmetic sequence, $u_3 = 11$ and $u_8 = 31$.

(a) Find the common difference. [2]

(b) Find the first term. [1]

(c) Find u_{20}. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Count the steps between the terms	$u_8 - u_3 = 5d = 20$	(M1)
	Or $u_1 + 2d = 11$ *and* $u_1 + 7d = 31$.	
Step 2 Solve for d	$d = 4$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Go back to u_1	$u_1 = 11 - 2 \times 4 = 3$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write $u_1 + 19d$	$u_{20} = 3 + 19 \times 4$	(M1)
Step 2 Compute	$= 79$	A1

B Arithmetic series

EXAM-STYLE QUESTION 2

Paper 1 · no calculator · [Maximum mark: 4]

Consider the series $\sum_{k=1}^{20}(3k+2)$.

(a) Write down the first term and the common difference. [2]

(b) Find the sum of the series. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Put $k=1$	$u_1 = 5$	A1
Step 2 Common difference: the coefficient of k	$d = 3$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute in S_n	$\frac{20}{2}(2\times 5 + 19\times 3)$ or $\frac{20}{2}(5+62)$	(M1)
Step 2 Compute	$= 670$	A1

EXAM-STYLE QUESTION 3

Paper 1 · no calculator · [Maximum mark: 4]

The odd numbers 1, 3, 5, … form an arithmetic sequence.

(a) Show that the sum of the first n odd numbers is n^2. [2]

(b) Hence find $1+3+5+\cdots+99$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Substitute $u_1 = 1, d = 2$ in S_n	$S_n = \frac{n}{2}(2\times 1 + (n-1)\times 2)$	M1
Step 2 Simplify	$= \frac{n}{2}\times 2n$	A1
Step 3 Reach the form given	$= n^2$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 How many terms? Solve $u_n = 99$	$99 = 2n-1$, so $n = 50$	(M1)
Step 2 Use the result just shown	$50^2 = 2500$	A1

C In context, with the GDC (Paper 2)

EXAM-STYLE QUESTION 4 Paper 2 · calculator · [Maximum mark: 7]

The first row of a theatre has 20 seats. Each row has 3 more seats than the row in front of it.

(a) Find the number of seats in the 15th row. [2]

(b) Find the total number of seats in the first 15 rows. [2]

(c) The theatre must have at least 1000 seats. Find the least number of rows it needs. [3]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 One row: u_{15}	$20 + 14 \times 3$	(M1)
Step 2 Compute	$= 62$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 All the rows: S_{15}	$\frac{15}{2}(20 + 62)$	(M1)
Step 2 Compute	$= 615$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the inequality with S_n	$\frac{n}{2}(40 + 3(n-1)) \geq 1000$	M1
Step 2 Solve on the GDC	$n \geq 20.4 \quad (20.379\ldots)$ *Or $S_{20} = 970$ and $S_{21} = 1050$.*	(A1)
Step 3 Next integer up	21 rows	A1

IV WHICH METHOD? READ THE QUESTION

1.2 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"arithmetic", "common difference"	$u_n = u_1 + (n-1)d$	u_1 and d first D1
two terms given	the steps between them	$u_m - u_k = (m-k)d$ P1
"is x a term?"	solve $u_n = x$	a positive integer n, or no W1
"the sum of the first n terms"	S_n	$\frac{n}{2}(2u_1 + (n-1)d)$ or $\frac{n}{2}(u_1 + u_n)$ D2
Σ	first term, d, number of terms	then S_n P2
one row or all the rows?	u_n or S_n	read the question twice W2
"the least n such that $S_n > \ldots$"	a quadratic in n	the GDC; the next integer up W2

In the exam 1 *command terms*

Show that: start from the formula, end at the expression given. **Hence**: use the result just shown. **Write down**: the value, no working needed.

V ON THE GDC: PAPER 2

SL 1.2 · NumWorks · every calculator move of this topic

1 A sequence on the GDC

Sequences → Explicit → u(n)=20+3(n-1) → Table

Enter u_n; the table lists the terms. For the sums, a second sequence $v(n) = \frac{n}{2}(40 + 3(n-1))$. **On paper:** $S_{20} = 970 < 1000$, $S_{21} = 1050$, so 21 rows.

SEQUENCES · TABLE

n	u(n)	sum
20	77	970
21	80	1050

ANSWERS TO THE APPLY QUESTIONS

I.1 -4; -6

I.2 13

I.3 3; 5

I.4 yes, u_{17}

I.5 yes, u_{29}

I.6 5; 4

I.7 4; 4

II.1 570

II.2 402

II.3 2550

II.4 21 terms; 840

II.5 180

II.6 1605

II.7 €38

II.8 €258

II.9 $n = 23$

II.10 10 or 11 (the 11th term is 0)

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