Arithmetic sequences and series

Skills sheet · SL AA 1.2

KEY FACTS

$u_n = u_1 + (n-1)d$ D1
Two terms: $u_m - u_k = (m-k)d$ P1
$S_n = \frac{n}{2}\big(2u_1 + (n-1)d\big) = \frac{n}{2}(u_1 + u_n)$ D2
$\sum$: first term, d, number of terms P2

$n-1$ steps to reach u_n W1.
One row is u_n, all rows S_n W2.
n must be a positive integer W1.

WHAT UNLOCKS

- "common difference" $\rightarrow$ u_1 and d.
- "is x a term?" $\rightarrow$ solve $u_n = x$, look for a whole n.
- "sum of ..." $\rightarrow$ S_n, with n counted carefully.
- "least n such that $S_n > \ldots$" $\rightarrow$ a quadratic, round up.

WORKED EXAMPLE ①

$u_1 = 7, d = 3$. **Find** u_{12}. no GDC

$7 + 11 \times 3$ (M1)
40 A1

Method
The nth term
① Write u_1 and d.
② $n-1$ steps of d.
③ Calculate.

WORKED EXAMPLE ②

Find S_{10} **for** $u_1 = 7, d = 3$. no GDC

$\frac{10}{2}(14 + 9 \times 3)$ (M1)
205 A1

Method
The sum of n terms
① Count the terms: n.
② Use $\frac{n}{2}(2u_1 + (n-1)d)$.
③ Or $\frac{n}{2}(u_1 + u_n)$.
A sum is not a term.

FLUENCY

●○○ **1.** 5, 9, 13, ...: d no GDC
●○○ **2.** 5, 9, 13, ...: u_{10} no GDC
●○○ **3.** 40, 37, 34, ...: u_n no GDC
●○○ **4.** $u_1 = 2, d = 5$: S_8 no GDC
●○○ **5.** $1 + 2 + \cdots + 100$ no GDC
●○○ **6.** Is 100 a term of 5, 9, 13, ...? no GDC

REASONING

●●● **Question 1.** In an arithmetic sequence, $u_5 = 23$ and $u_{12} = 58$. [no GDC]

(a) Find d and u_1. [3]

●●● **Question 2.** Consider $\sum_{k=1}^{15}(4k-1)$. [no GDC]

(a) Find the sum. [2]

●●● **Question 3.** An arithmetic sequence has $u_1 = 50$ and $d = -4$. [no GDC]

(a) Find the first negative term. [2]

PROBLEMS

●●● **Question 4.** The sum of the first n terms of an arithmetic sequence is $S_n = 3n^2 + 2n$. [no GDC]

(a) Find u_1 and d. [3]

●●● **Question 5.** A worker earns 30 000 in the first year, with a rise of 1500 each year. [with GDC]

(a) Find the total earned in the first 10 years. [2]

(b) Find the year in which the total first exceeds 500 000. [2]

ANSWERS

1 4

2 41

3 $43 - 3n$

4 $\frac{8}{2}(4 + 35) = 156$

5 5050

6 $4n + 1 = 100$, $n = 23.75$: no

Q1 $7d = 35$, $d = 5$, $u_1 = 3$

Q2 $u_1 = 3$, $u_{15} = 59$, $\frac{15}{2}(62) = 465$

Q3 $50 - 4(n - 1) < 0$, $n > 13.5$: $u_{14} = -2$

Q4 $u_1 = S_1 = 5$, $S_2 = 16$, $u_2 = 11$, $d = 6$

Q5 (a) $5(60\,000 + 13\,500) = 367\,500$ (b) $S_{13} = 507\,000$, $S_{12} = 459\,000$: the 13th year

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