Exponents and logarithms

Skills sheet · SL AA 1.5 · 1.7

KEY FACTS

$a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$ P1

$a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ D1

$\log_a x = y \iff a^y = x$ D2

$\log(xy) = \log x + \log y$, $\log x^m = m \log x$, $\log_a x = \frac{\ln x}{\ln a}$ P2

Same base first W1.

$\log(x+y)$ *does not split* W2.

Reject a negative argument W2.

WHAT UNLOCKS

- "as a power of 2" $\rightarrow$ $4 = 2^2$, $8 = 2^3$, then the laws.
- unknown exponent $\rightarrow$ logarithms: $x = \frac{\ln b}{\ln a}$.
- $\log + \log = k$ $\rightarrow$ one log, then a^k.
- "in terms of p, q" $\rightarrow$ split, then substitute.

WORKED EXAMPLE ①

Simplify $\dfrac{8^{\frac{2}{3}} \times 2^{-1}}{4^{\frac{1}{2}}}$. no GDC

$\dfrac{4 \times \frac{1}{2}}{2}$ (M1)

1 A1

Method

Rational exponents

① Root first: $\sqrt[3]{8} = 2$.
② Then the power.
③ Negative: the reciprocal.

WORKED EXAMPLE ②

Solve $5^x = 40$. with GDC

$x = \dfrac{\ln 40}{\ln 5}$ (M1)

2.29 A1

Method

An exponential equation

① Take ln of both sides.
② Bring the exponent down.
③ Divide.

$\ln 40 / \ln 5$, ***not*** $\ln 8$.

FLUENCY

●○○ **1.** $32^{\frac{3}{5}}$ no GDC

●○○ **2.** $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ no GDC

●○○ **3.** $\log_2 64$ no GDC

●○○ **4.** $\log_{10} 0.001$ no GDC

●○○ **5.** $\log_3 12 - \log_3 4$ no GDC

●○○ **6.** $2 \ln e^3$ no GDC

REASONING

●●● **Question 1.** Solve $2^{3x-1} = 16^x$. [no GDC]

(a) Find x. [2]

●●● **Question 2.** Solve $\log_5(2x+1) - \log_5(x-1) = 1$. [no GDC]

(a) Find x. [3]

●●● **Question 3.** Let $\ln a = p$ and $\ln b = q$. [no GDC]

(a) Write $\ln \sqrt{ab^3}$ in terms of p and q. [2]

PROBLEMS

●●● **Question 4.** Solve $4^x - 6 \times 2^x + 8 = 0$. [no GDC]

(a) Find both solutions. [4]

●●● **Question 5.** A sum doubles every 6 years. [with GDC]

(a) Find the time for it to be multiplied by 10. [2]

ANSWERS

1 8

2 $\frac{3}{2}$

3 6

4 -3

5 1

6 6

Q1 $3x - 1 = 4x$, $x = -1$

Q2 $\frac{2x+1}{x-1} = 5$, $x = 2$

Q3 $\frac{1}{2}(p + 3q)$

Q4 $(2^x)^2 - 6(2^x) + 8 = 0$: $2^x = 2$ or 4, $x = 1$ or 2

Q5 $2^{t/6} = 10$: $t = 6\frac{\ln 10}{\ln 2} = 19.9$ years

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