Geometric sequences and series

Skills sheet · SL AA 1.3 · 1.8

KEY FACTS

$u_n = u_1 r^{n-1}$ D1

Two terms: $\dfrac{u_m}{u_k} = r^{m-k}$ P1

$S_n = \dfrac{u_1(r^n - 1)}{r - 1}$ D1

$S_\infty = \dfrac{u_1}{1 - r}$, only when $|r| < 1$ D2

Divide to find r W1.

$(-2)^{n-1}$, with brackets W1.

Say "$|r| < 1$" first W2.

WHAT UNLOCKS

- "common ratio" $\rightarrow$ $r = \frac{u_{n+1}}{u_n}$.
- "sum to infinity" $\rightarrow$ check $|r| < 1$, then $\frac{u_1}{1-r}$.
- "for which x" $\rightarrow$ $-1 < r < 1$, solved for x.
- "least n" $\rightarrow$ logarithms or the GDC table.

WORKED EXAMPLE ①

3, 6, 12, … **Find** u_8. no GDC

3×2^7 (M1)

384 A1

Method

The nth term

① Find r by dividing.
② Power $n - 1$.
③ Calculate.

WORKED EXAMPLE ②

Find S_∞ **of** $8 + 4 + 2 + \cdots$ no GDC

$r = \frac{1}{2}$, $|r| < 1$ R1

$\dfrac{8}{1 - \frac{1}{2}} = 16$ A1

Method

A sum to infinity

① Find r.
② State $|r| < 1$.
③ $\frac{u_1}{1-r}$.

No S_∞ when $|r| \geq 1$.

FLUENCY

●○○ **1.** 2, 6, 18, …: r no GDC

●○○ **2.** 2, 6, 18, …: u_6 no GDC

●○○ **3.** 64, −32, 16, …: r no GDC

●○○ **4.** $u_1 = 1$, $r = 2$: S_{10} no GDC

●○○ **5.** S_∞ of $9 + 3 + 1 + \cdots$ no GDC

●○○ **6.** Does $1 + 1.1 + 1.21 + \cdots$ have a sum to infinity? no GDC

REASONING

●●● **Question 1.** A geometric sequence has $u_3 = 20$ and $u_5 = 80$, with $r > 0$. [no GDC]

(a) Find r and u_1. [3]

●●● **Question 2.** A geometric series has $u_1 = 12$ and $S_\infty = 48$. [no GDC]

(a) Find r. [2]

●●● **Question 3.** Consider $1 + \frac{x}{3} + \frac{x^2}{9} + \cdots$ [no GDC]

(a) Find the values of x for which the sum to infinity exists. [2]

PROBLEMS

●●● **Question 4.** A geometric sequence has $u_1 = 3$ and $r = 1.5$. [with GDC]

(a) Find the least n for which $S_n > 1000$. [3]

●●● **Question 5.** The terms k, $k + 6$, $4k$ are consecutive terms of a geometric sequence, $k > 0$. [no GDC]

(a) Find k. [3]

ANSWERS

1 3

2 $2 \times 3^5 = 486$

3 $-\frac{1}{2}$

4 1023

5 $\frac{9}{1-\frac{1}{3}} = 13.5$

6 no: $r = 1.1 > 1$

Q1 $r^2 = 4$, $r = 2$, $u_1 = 5$

Q2 $\frac{12}{1-r} = 48$, $r = \frac{3}{4}$

Q3 $\left|\frac{x}{3}\right| < 1$: $-3 < x < 3$

Q4 $\frac{3(1.5^n-1)}{0.5} > 1000$: $1.5^n > 167.67$, $n > 12.6$, $n = 13$

Q5 $(k+6)^2 = 4k^2$, $k + 6 = 2k$, $k = 6$

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