Standard form and proof

Two habits that the whole course relies on. Standard form writes the very large and the very small so that they can be compared and multiplied at a glance. A proof shows that a statement is true for every value, not just for the ones we tried.

I STANDARD FORM

SL 1.1 · $a \times 10^k$, $1 \leq a < 10$, $k \in \mathbb{Z}$; operations

I.1 Writing a number in standard form

Definition 1 *standard form*

$a \times 10^k$, with $1 \leq a < 10$ and k an integer. k counts the places the decimal point moves: $k > 0$ for large numbers, $k < 0$ for numbers between 0 and 1.

Watch out 1

12×10^3 and 0.45×10^5 are not in standard form: the first factor must be at least 1 and less than 10. The GDC writes 1.2E4: on paper, 1.2×10^4.

Example 1

Write 2 040 000 and 0.000 63 in standard form.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Place the point after the first non-zero digit	2.04 and 6.3
Step 2 Count the places it moved	6 to the left; 4 to the right
Step 3 Large: $k > 0$; small: $k < 0$	2.04×10^6 and 6.3×10^{-4}

APPLY

1 Write 38 500 in standard form.

2 Write 0.0902 in standard form.

3 Write 7.1×10^5 as an ordinary number.

4 Write 25×10^3 in standard form.

I.2 Multiplying and dividing

Property 1 *multiplying and dividing*

> Multiply (or divide) the numbers a, add (or subtract) the powers of 10, then adjust so that the first factor is between 1 and 10.

Example 2

Calculate $(2.5 \times 10^3) \times (6 \times 10^{-5})$, in standard form.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Multiply the numbers	$2.5 \times 6 = 15$
Step 2 Add the powers	$10^{3+(-5)} = 10^{-2}$
Step 3 Adjust the first factor $15 = 1.5 \times 10$: the power goes up by one.	$15 \times 10^{-2} = 1.5 \times 10^{-1}$

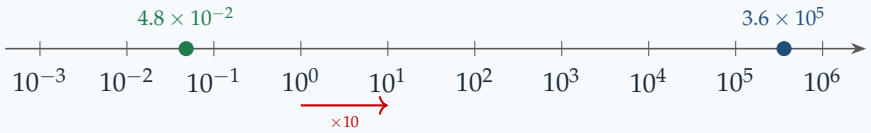


Figure 1 — Exam-style question 1 on the scale: $3.6 \times 10^5 = 360\,000$ between 10^5 and 10^6; $4.8 \times 10^{-2} = 0.048$ between 10^{-2} and 10^{-1}.

APPLY

5 $(7 \times 10^4) \times (3 \times 10^2)$

6 $\dfrac{9.6 \times 10^7}{1.2 \times 10^3}$

7 $\dfrac{4.2 \times 10^{-3}}{6 \times 10^5}$

8 $(1.5 \times 10^6)^2$

I.3 Adding and subtracting

Example 3

Calculate $3.2 \times 10^5 + 4.5 \times 10^4$, in standard form.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write both with the same power Or as ordinary numbers: $320\,000 + 45\,000$.	$32 \times 10^4 + 4.5 \times 10^4$
Step 2 Add the numbers	36.5×10^4
Step 3 Adjust the first factor	3.65×10^5

APPLY

9 $6 \times 10^{-3} + 7 \times 10^{-4}$

10 $5.1 \times 10^{6} - 2.4 \times 10^{5}$

I.4 Standard form in context

Example 4

A hard drive holds 2×10^{12} bytes. One photo takes 4.8×10^{6} bytes. How many whole photos fit on the drive?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the division	$\dfrac{2 \times 10^{12}}{4.8 \times 10^{6}}$
Step 2 Compute (GDC)	$416\,666.6\ldots$
Step 3 Answer the question: whole photos Round down: the last photo does not fit.	$416\,666$ photos $(\approx 4.17 \times 10^{5})$

APPLY

11 A light-year is 9.46×10^{12} km. How many km are 4.2 light-years?

12 A grain of sand weighs 6.5×10^{-4} g. How many grains in 1 kg?

II SIMPLE DEDUCTIVE PROOF

SL 1.6 · identities, ≡, LHS to RHS, even and odd integers

II.1 Proving an identity: LHS to RHS

Definition 2 *identity*

An **identity** is true for every value of the variable, written with ≡. To **prove** it, start from one side (the more complicated one, the LHS) and transform it, step by step, into the other side.

Watch out 2

Do not work on both sides at once, and do not start from what you want to prove. Checking a few values is not a proof. End with a sentence.

Example 5

Prove that $(a+b)^2-(a-b)^2 \equiv 4ab$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Start from the LHS	$\text{LHS} = (a+b)^2-(a-b)^2$
Step 2 Expand each bracket Keep the bracket after the minus sign.	$= a^2+2ab+b^2-(a^2-2ab+b^2)$
Step 3 Simplify	$= 4ab$
Step 4 Conclude	$= \text{RHS}$

APPLY

1 Prove that $(x+5)(x-2)-x(x+3) \equiv -10$.

2 Prove that $(2x-1)^2+4x \equiv 4x^2+1$.

II.2 Even and odd numbers

Property 2 *writing integers*

n an integer: $2n$ is any even number, $2n+1$ any odd number; n, $n+1$, $n+2$ are consecutive integers. A multiple of k is $k \times$ (an integer).

Example 6

Prove that the product of two odd numbers is odd.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the two odd numbers Two letters: the numbers need not be equal.	$2m+1$ and $2n+1$, m, n integers
Step 2 Multiply out	$4mn+2m+2n+1$
Step 3 Show the form $2(\ldots)+1$	$= 2(2mn+m+n)+1$
Step 4 Conclude	$2mn+m+n$ is an integer, so the product is odd

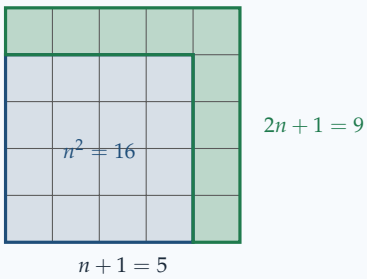


Figure 2 — Why $(n+1)^2 - n^2 = 2n + 1$, an odd number: from the $n \times n$ square to the $(n+1) \times (n+1)$ square, an L-shape of $n + n + 1$ unit squares. Here $n = 4$: $25 - 16 = 9$.

APPLY

3 Prove that an even number plus an odd number is odd.

4 Prove that the sum of four consecutive integers is even.

II.3 Showing a multiple of k

Example 7

Let n be an integer. Show that $(3n+1)^2 - (3n-1)^2$ is a multiple of 12.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Expand each bracket	$9n^2 + 6n + 1 - (9n^2 - 6n + 1)$
Step 2 Simplify	$= 12n$
Step 3 Write $k\times$(an integer), conclude	$12 \times n$, n an integer: a multiple of 12

APPLY

5 Show that $6n + 9$ is a multiple of 3.

6 Show that $(n+5)^2 - (n+1)^2$ is a multiple of 8.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Standard form

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 5]

Let $A = 3.6 \times 10^5$ and $B = 4.8 \times 10^{-2}$.

(a) Write 0.000 52 in standard form. [1]

(b) Find AB, giving your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [2]

(c) Find $\frac{A}{B}$, giving your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Point after the first digit, count the places	5.2×10^{-4}	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Multiply the numbers, add the powers	$3.6 \times 4.8 \times 10^{5-2} = 17.28 \times 10^3$	(M1)
Step 2 Adjust the first factor	1.728×10^4 *Accept* 1.73×10^4.	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Divide the numbers, subtract the powers	$\frac{3.6}{4.8} \times 10^{5+2} = 0.75 \times 10^7$	(M1)
Step 2 Adjust the first factor	7.5×10^6	A1

B Proof

EXAM-STYLE QUESTION 2

Paper 1 · no calculator · [Maximum mark: 4]

Let n be an integer.

(a) Show that $(n+3)^2-(n-1)^2 \equiv 8(n+1)$. [3]

(b) Hence explain why the difference between the squares of two integers that differ by 4 is always a multiple of 8. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Expand each bracket	$(n+3)^2 = n^2+6n+9$ and $(n-1)^2 = n^2-2n+1$	M1A1
	M1 for expanding a bracket; A1 for both correct.	
Step 2 Simplify	$n^2+6n+9-n^2+2n-1 = 8n+8$	A1
	The minus sign applied to every term of the second bracket.	
Step 3 Factorise: the form given	$= 8(n+1)$	AG

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write $8\times$(an integer), conclude	$n+1$ is an integer, so $8(n+1)$ is a multiple of 8	R1
	The two integers are $n+3$ and $n-1$.	

EXAM-STYLE QUESTION 3

Paper 1 · no calculator · [Maximum mark: 4]

In this question, n is an integer.

(a) Show that the sum of three consecutive integers is a multiple of 3. [2]

(b) Show that the sum of two consecutive odd numbers is a multiple of 4. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write three consecutive integers, add	$n+(n+1)+(n+2) = 3n+3$	M1
Step 2 Factorise, conclude	$= 3(n+1)$, a multiple of 3	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write two consecutive odd numbers, add	$(2n+1)+(2n+3) = 4n+4$	M1
	Two consecutive odd numbers: $2n+1$ and $2n+3$.	
Step 2 Factorise, conclude	$= 4(n+1)$, a multiple of 4	A1

C In context, with the GDC (Paper 2)

EXAM-STYLE QUESTION 4 Paper 2 · calculator · [Maximum mark: 6]

The mean distance from the Earth to the Sun is 1.496×10^8 km. Light travels at 3.00×10^5 km per second.

(a) Find the time, in seconds, that light takes to travel from the Sun to the Earth. Give your answer in standard form. [2]

(b) Assume the Earth moves around the Sun on a circle of radius 1.496×10^8 km. Find the length of one orbit, in standard form. [2]

(c) One orbit takes 365.25 days. Find the mean speed of the Earth, in km per hour, in standard form. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Time = distance ÷ speed	$\dfrac{1.496 \times 10^8}{3 \times 10^5}$	(M1)
Step 2 Compute, standard form	4.99×10^2 s $(498.666\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Circumference $2\pi r$	$2\pi \times 1.496 \times 10^8$	(M1)
Step 2 Compute, standard form	9.40×10^8 km $(9.39964\ldots \times 10^8)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Divide by the hours in a year	$\dfrac{9.39964\ldots \times 10^8}{365.25 \times 24}$	(M1)
	$365.25 \times 24 = 8766$ *hours.*	
Step 2 Compute, standard form	1.07×10^5 km/h $(107\,228\ldots)$	A1

IV WHICH METHOD? READ THE QUESTION

1.6 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"in the form $a \times 10^k$"	$1 \le a < 10$, k an integer	adjust a, change k to match D1
a product or a quotient	a's together, powers together	multiply the a's, add the powers; then adjust P1
the GDC shows E	Ek means $\times 10^k$	write $\times 10^k$ on paper W1
"show that … ≡ …"	start from one side	LHS, steps, = RHS; never both sides at once D2
"even", "odd", "consecutive"	$2n$, $2n+1$, n and $n+1$	one letter, an integer P2
"is a multiple of k"	factor out k	$k \times (\ldots)$ and say the bracket is an integer P2
"hence explain"	the result just proved	one sentence that uses it W2

In the exam 1 *command terms*

Show that: every step written, from the given expression to the result. **Hence**: use what was just shown. **Standard form**: $a \times 10^k$ with $1 \le a < 10$.

V ON THE GDC: PAPER 2

SL 1.1 · 1.6 · NumWorks · every calculator move of this topic

1 Standard form on the GDC

Settings → Result format → Scientific

Type $\times 10^k$ with the $\times 10^x$ key (shown E). The scientific format gives every result in standard form. **On paper:** 9.40×10^8, never 9.40E8.

SETTINGS · RESULT FORMAT

```
3.6E5*4.8E-2
1.728E4
2*pi*1.496E8
9.399645E8
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ANSWERS TO THE APPLY QUESTIONS

I.1 3.85×10^4

I.2 9.02×10^{-2}

I.3 710 000

I.4 2.5×10^4

I.5 2.1×10^7

I.6 8×10^4

I.7 7×10^{-9}

I.8 2.25×10^{12}

I.9 6.7×10^{-3}

I.10 4.86×10^6

I.11 3.97×10^{13} km

I.12 1.54×10^6

II.1 $x^2 + 3x - 10 - x^2 - 3x = -10$

II.2 $4x^2 - 4x + 1 + 4x = 4x^2 + 1$

II.3 $2m + 2n + 1 = 2(m + n) + 1$

II.4 $4n + 6 = 2(2n + 3)$

II.5 $3(2n + 3)$

II.6 $8n + 24 = 8(n + 3)$

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