The binomial theorem

Skills sheet · SL AA 1.9

KEY FACTS

$\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$, $\binom{n}{r} = \binom{n}{n-r}$ D1

Pascal: each entry the sum of the two above; row sum 2^n P1

$(a+b)^n = \sum \binom{n}{r} a^{n-r} b^r$ P2

One term: the general term, then r P3

Row n starts at row 0 W1.

$(3x)^3 = 27x^3$ W2.

Coefficient: no x W2.

WHAT UNLOCKS

- "expand" $\rightarrow$ row n, every term simplified.
- "the term in x^k" $\rightarrow$ general term, power of x, solve for r.
- "constant term" $\rightarrow$ power of x equal to 0.
- a product with $(1+ax)$ $\rightarrow$ two ways to make the power.

WORKED EXAMPLE ①

Find $\binom{7}{3}$. no GDC

$\dfrac{7 \times 6 \times 5}{3 \times 2 \times 1}$ (M1)

35 A1

Method

A binomial coefficient

① r factors on top, from n down.
② Divide by $r!$.
③ Check with Pascal.

WORKED EXAMPLE ②

Find the coefficient of x^2 in $(1+2x)^5$. no GDC

$\binom{5}{2}(2x)^2 = 10 \times 4x^2$ (M1)

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Method

One coefficient

① General term with brackets.
② Choose r for the power.
③ Only the number.

$(2x)^2$, ***not*** $2x^2$.

FLUENCY

●○○ **1.** $\binom{5}{0}$ no GDC

●○○ **2.** $\binom{5}{2}$ no GDC

●○○ **3.** $\binom{8}{7}$ no GDC

●○○ **4.** Row 5 of Pascal's triangle no GDC

●○○ **5.** $(a+b)^3$ no GDC

●○○ **6.** Sum of row 8 no GDC

REASONING

●●● **Question 1.** Expand $(1-2x)^4$. [no GDC]

(a) Write every term. [3]

●●● **Question 2.** Consider $(x+3)^7$. [no GDC]

(a) Find the coefficient of x^5. [2]

●●● **Question 3.** Consider $\left(2x-\frac{1}{x}\right)^4$. [no GDC]

(a) Find the constant term. [3]

PROBLEMS

●●● **Question 4.** The coefficient of x^2 in $(1+kx)^6$ is 60, with $k>0$. [no GDC]

(a) Find k. [3]

●●● **Question 5.** Consider $(1+x)^5(2-x)$. [no GDC]

(a) Find the coefficient of x^3. [3]

ANSWERS

1 1

2 10

3 8

4 1 5 10 10 5 1

5 $a^3 + 3a^2b + 3ab^2 + b^3$

6 256

Q1 $1 - 8x + 24x^2 - 32x^3 + 16x^4$

Q2 $\binom{7}{2} \times 3^2 = 189$

Q3 $r = 2$: $\binom{4}{2}(2x)^2\left(-\frac{1}{x}\right)^2 = 24$

Q4 $15k^2 = 60$, $k = 2$

Q5 $2 \times \binom{5}{3} - \binom{5}{2} = 20 - 10 = 10$

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